

# Languages & Notations for Systems Biology

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2004-09-15 Unconventional Programming Paradigms '04  
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# Aims

Modeling **biological systems**.

Helping out in Systems Biology.

By adapting paradigms and techniques developed for modeling **information-processing systems**.

Because they have some similar features:

Deep layering of abstractions.

Complex composition of simpler components.

Discrete (non-linear) evolution.

Digital coding of information.

Reactive information-driven behavior.

Very high degree of concurrency.

'Emergent behavior' (not obvious from part list).

# Methods

## Model Construction (writing things down precisely)

Studying the notations used in systems biology.

Formulating process calculi, for various purposes.

Studying their dynamics (semantics).

## Model Validation (using models for postdiction and prediction)

### Stochastic Simulation

Stochastic = Quantitative concurrent semantics.

Based on compositional descriptions.

### “Program” Analysis

Control flow analysis

Causality analysis

### Modelchecking

Standard, Quantitative, Probabilistic

# Storing Processes

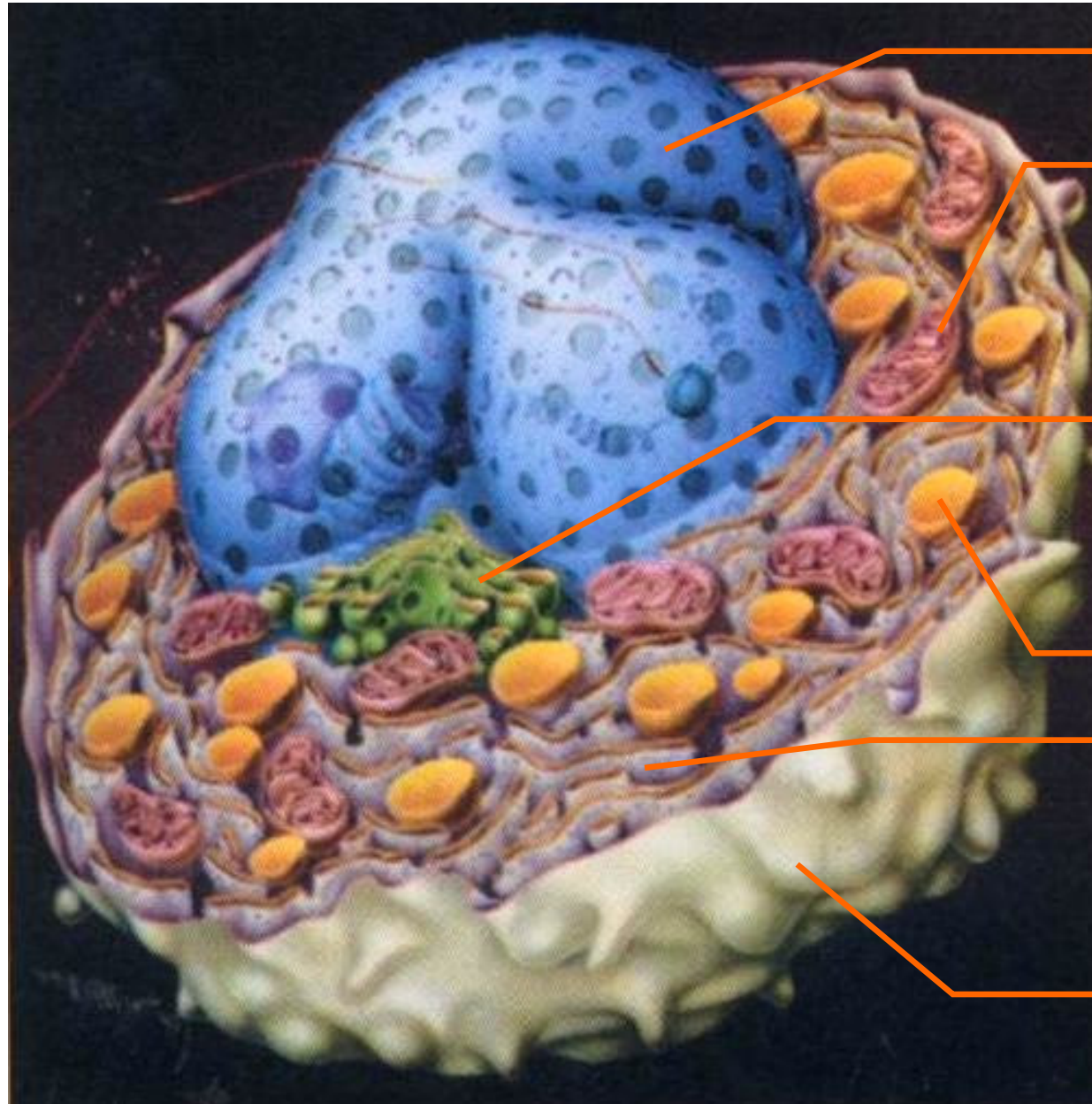
- Today we represent, store, and analyze:
  - Gene sequence data
  - Protein structure data
  - Metabolic network data
  - Compartmentalized reaction data (SBML)
  - ...
- How can we represent, store, and analyze *biological processes*?
  - Scalable, precise, dynamic, highly structured, maintainable representations for *systems biology*.
  - Not just huge lists of chemical reactions or differential equations.

# Structural Architecture

## Eukaryotic Cell

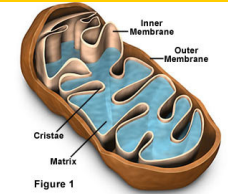
(10~100 trillion in human body)

Membranes everywhere

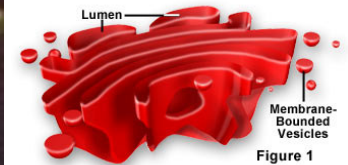


Nuclear membrane

Mitochondria

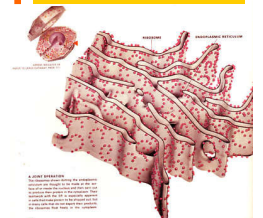


Golgi



Vesicles

E.R.



Plasma membrane (<10% of all membranes)

# Functional Architecture

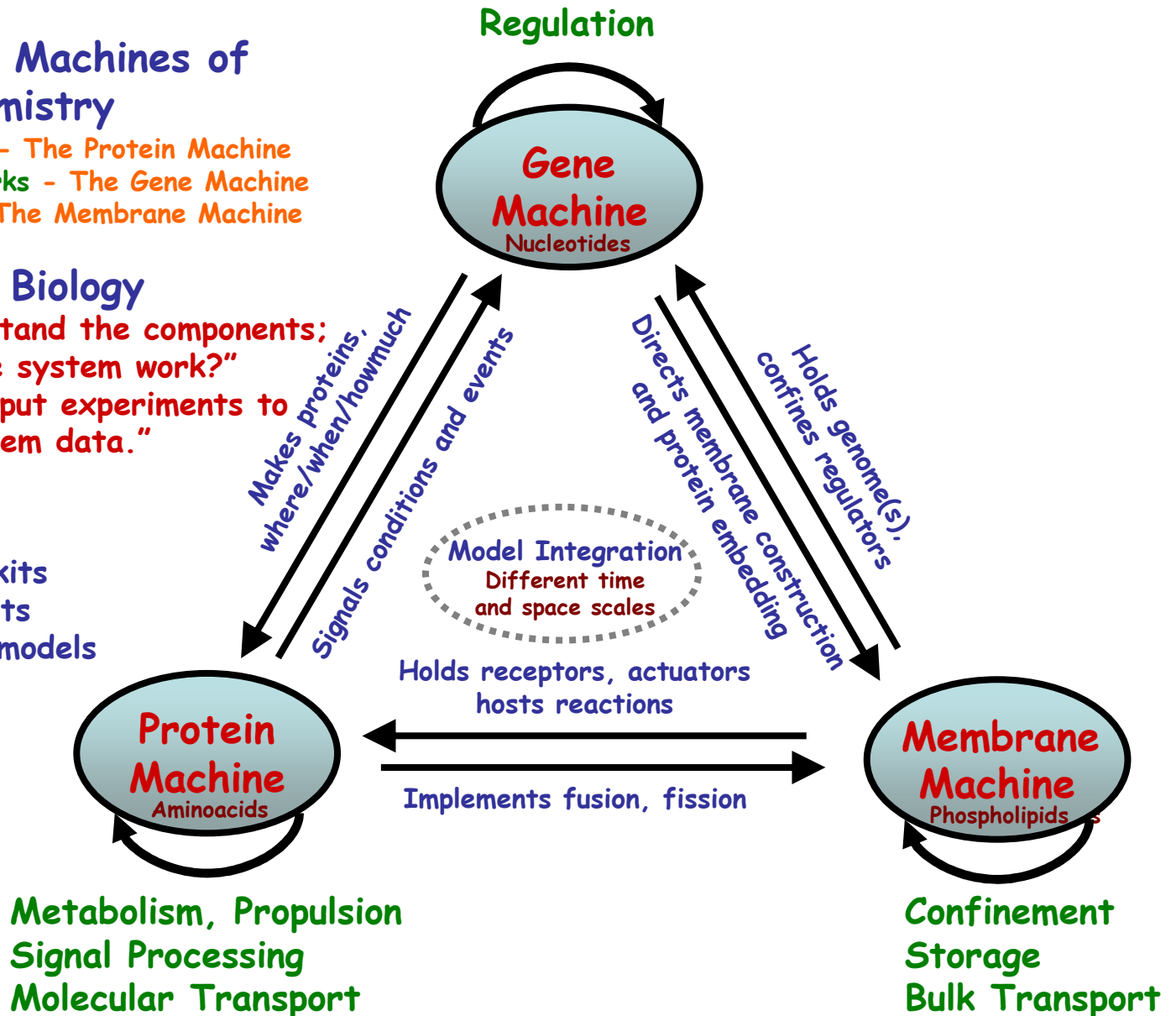
## The Abstract Machines of Biochemistry

Biochemical Networks - The Protein Machine  
 Gene Regulatory Networks - The Gene Machine  
 Transport Networks - The Membrane Machine

## Systems Biology

1. "We (kind of) understand the components; but how does the system work?"
2. "Use high-throughput experiments to gather system data."

Different chemical toolkits  
 Different instruction sets  
 Different programming models  
 Different notations

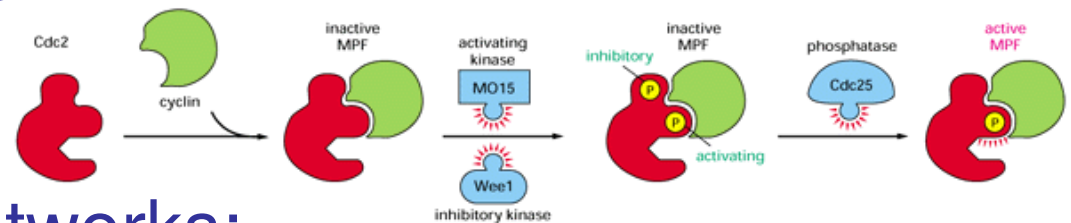




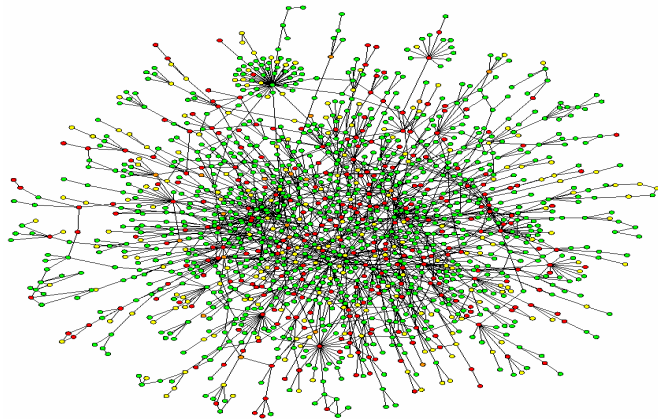
# 1. The Protein Machine

*Very close to the atoms.*

- Complex folded-up shapes that:
  - Fit together, dock, undock.
  - Excite/unexcite, warp each other.
  - Bring together, catalyze, transform materials.
  - Form complex aggregates and networks.



- Mapping out such networks:
  - In principle, it's “just” a very large set of chemical equations.
  - Notations have been developed to summarize and abstract.

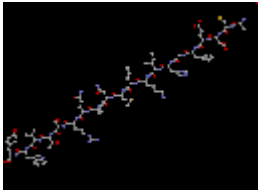


**An actual molecular interaction network.**

(Nodes are distinct protein kinds,  
arcs mean that two kinds of proteins interact.)

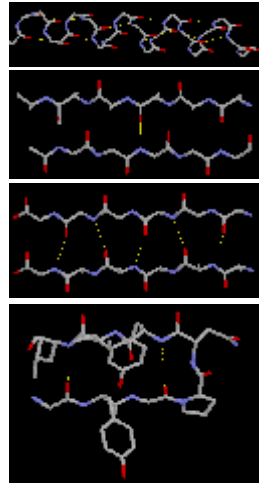
# Protein Structure

Primary



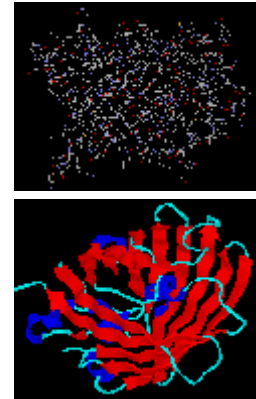
The 20 Aminoacids

Secondary



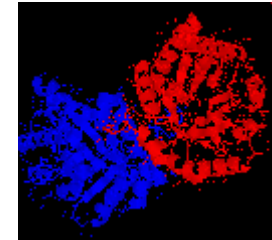
Alpha Helix, Beta Sheet

Tertiary



Green Fluorescent Protein

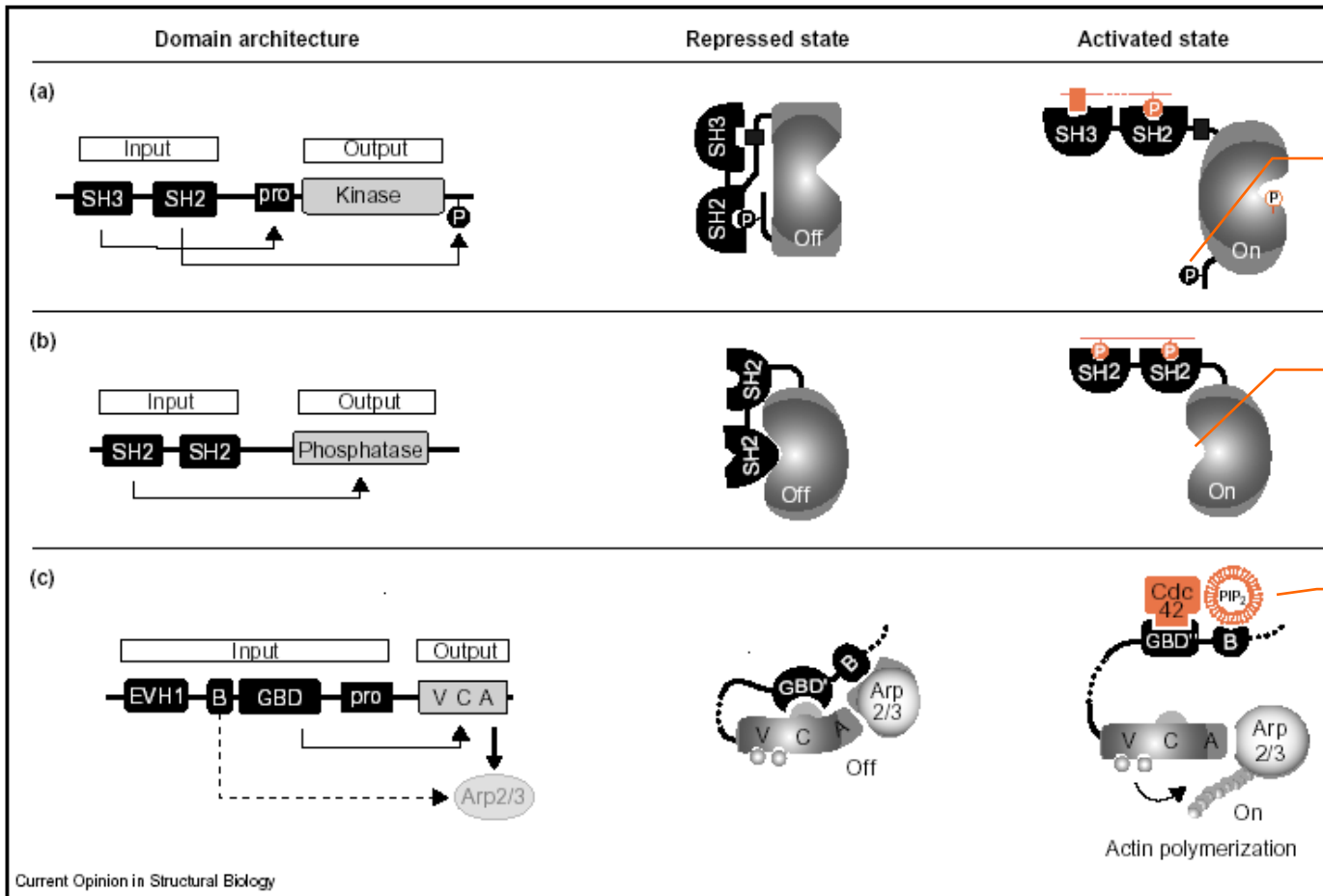
Quaternary



Triose Phosphate Isomerase



# Some Allosteric Switches



Allosteric ("other shape") reactions modify accessibility.

## Kinase

= donates phosphate P  
= phosphorylates other proteins

## Phosphatase

= accepts phosphate P  
= dephosphorylates other proteins

## Logical AND

at equal concentrations of the individual input stimuli, activation is much higher if both stimuli are present

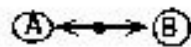
*"Phosphatase Kinase Kinase" = a kinase that activates a kinase that activates a phosphatase that deactivates a protein.*

Domain architecture and autoinhibitory interactions in modular switch proteins. (a) Src family kinases contain N-terminal SH3 and SH2 domains, and a kinase domain flanked by intramolecular SH3-binding and SH2-binding sites (when the C-terminal motif tyrosine is phosphorylated by Csk). The crystal structures of several family members show that both intramolecular domain interactions function in concert to lock the kinase in an inactive conformation. Activating stimuli (red) include external SH2 or SH3 ligands. After initial activation, the kinase is maintained in an active state by autophosphorylation of its activation loop. (b) SHP-2 phosphatase contains two SH2 domains and a phosphatase domain. The crystal structure of the phosphatase

shows that the N-terminal SH2 domain participates in an autoinhibitory interaction that directly blocks the phosphatase active site. Binding of external SH2 ligands activates by disrupting the autoinhibitory interaction. (c) N-WASP contains an Enabled VASP homology 1 (EVH1) domain, a B motif, a GBD, a proline-rich segment (pro) and an output region (VCA) that alone binds the Arp2/3 complex and stimulates its actin nucleation activity. The B and GBD motifs are required to repress activity and, by current models, are thought to participate in intracomplex interactions (only the structure of the GBD intramolecular complex for WASP is known). GTP-bound Cdc42 and PIP<sub>2</sub> synergistically activate N-WASP.

Humans have the same number of modular protein domains (building blocks) as worms, but twice the number of multi-domain proteins.

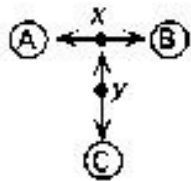
# MIM: Molecular Interaction Maps (Kohn)



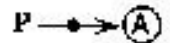
The double-headed line indicates that proteins **A** and **B** can bind to each other. The "node" placed on the line represents the **A:B** complex.



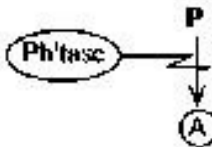
Asymmetric binding where protein **A** donates a peptide that binds to a receptor site or pocket on protein **B**.



Representation of multimolecular complexes:  $x$  is **A:B**;  $y$  is **(A:B):C**. This notation is extensible to any number of components in a complex.



Covalent modification of protein **A**. The single-headed line indicates that **A** can exist in a phosphorylated state. The node represents the phosphorylated species.



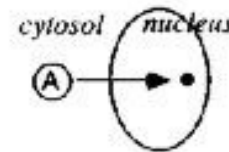
Cleavage of a covalent bond: dephosphorylation of **A** by a phosphatase.



Proteolytic cleavage at a specific site within a protein.



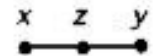
Stoichiometric conversion of **A** into **B**.



Transport of **A** from cytosol to nucleus. The node represents **A** after it has been transported into the nucleus.



Formation of a homodimer. Filled circle on the right represents another copy of **A**. The node on the line represents the homodimer **A:A**.



$z$  is the combination of states defined by  $x$  and  $y$ .



Enzymatic stimulation of a reaction.



General symbol for stimulation.



A bar behind the arrowhead signifies necessity.



General symbol for inhibition.



Shorthand symbol for transcriptional activation.



Shorthand symbol for transcriptional inhibition.



Degradation products

Taken from  
Kurt W. Kohn

# Molecular Interaction Maps

<http://www.cds.caltech.edu/~hsauro/index.htm>

JDesigner

## The p53-Mdm2 and DNA Repair Regulatory Network

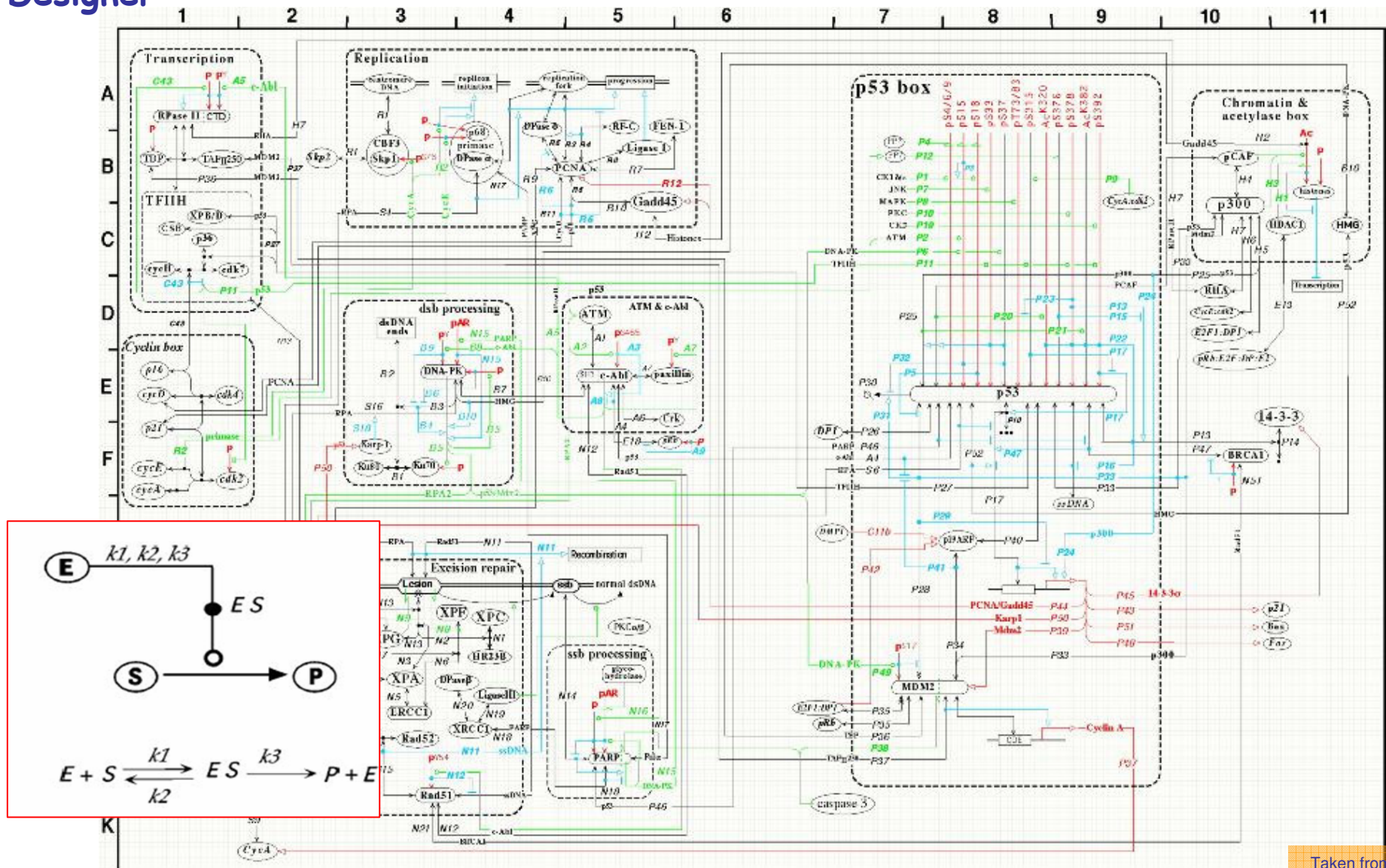


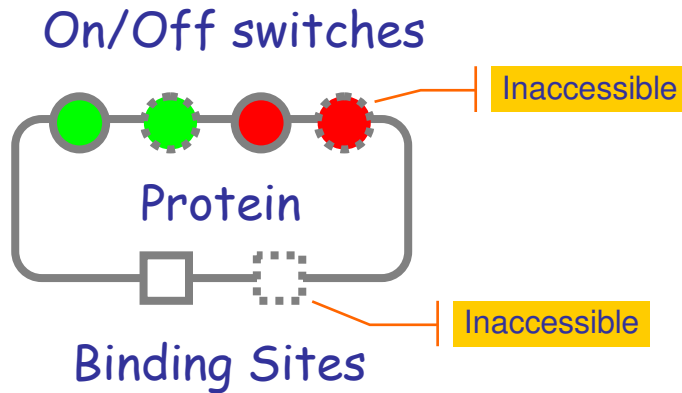
Figure 6B: The p53-Mdm2 and DNA repair regulatory network (version 2p - May 19, 1999)

Taken from  
Kurt W. Kohn

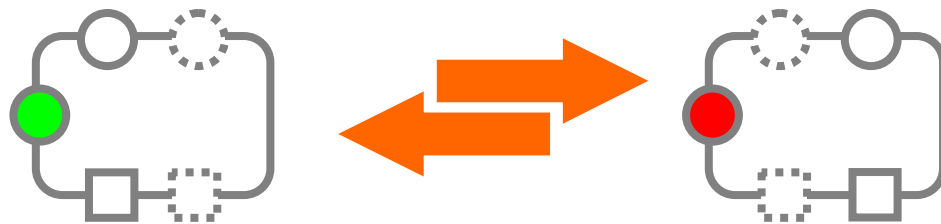
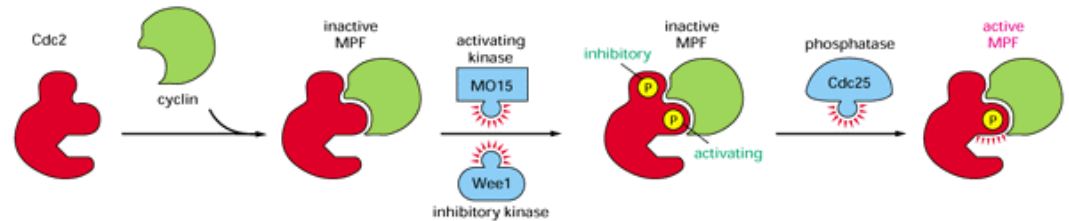


# The Protein Machine "Instruction Set"

cf. BioCalculus [Kitano&Nagasaki],  $\kappa$ -calculus [Danos&Laneve]

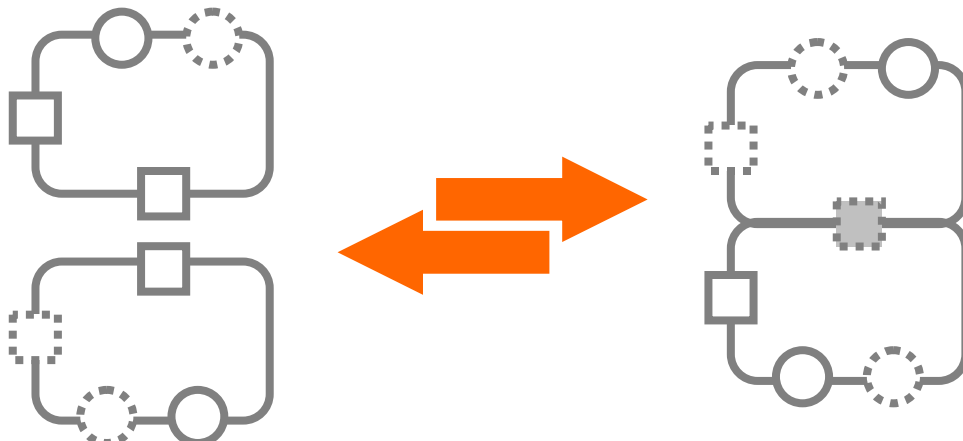


Each protein has a structure of binary switches and binding sites. But not all may be always *accessible*.



Switching of accessible switches.

- May cause other switches and binding sites to become (in)accessible.
- May be triggered or inhibited by nearby specific proteins in specific states.



Binding on accessible sites.

- May cause other switches and binding sites to become (in)accessible.
- May be triggered or inhibited by nearby specific proteins in specific states.

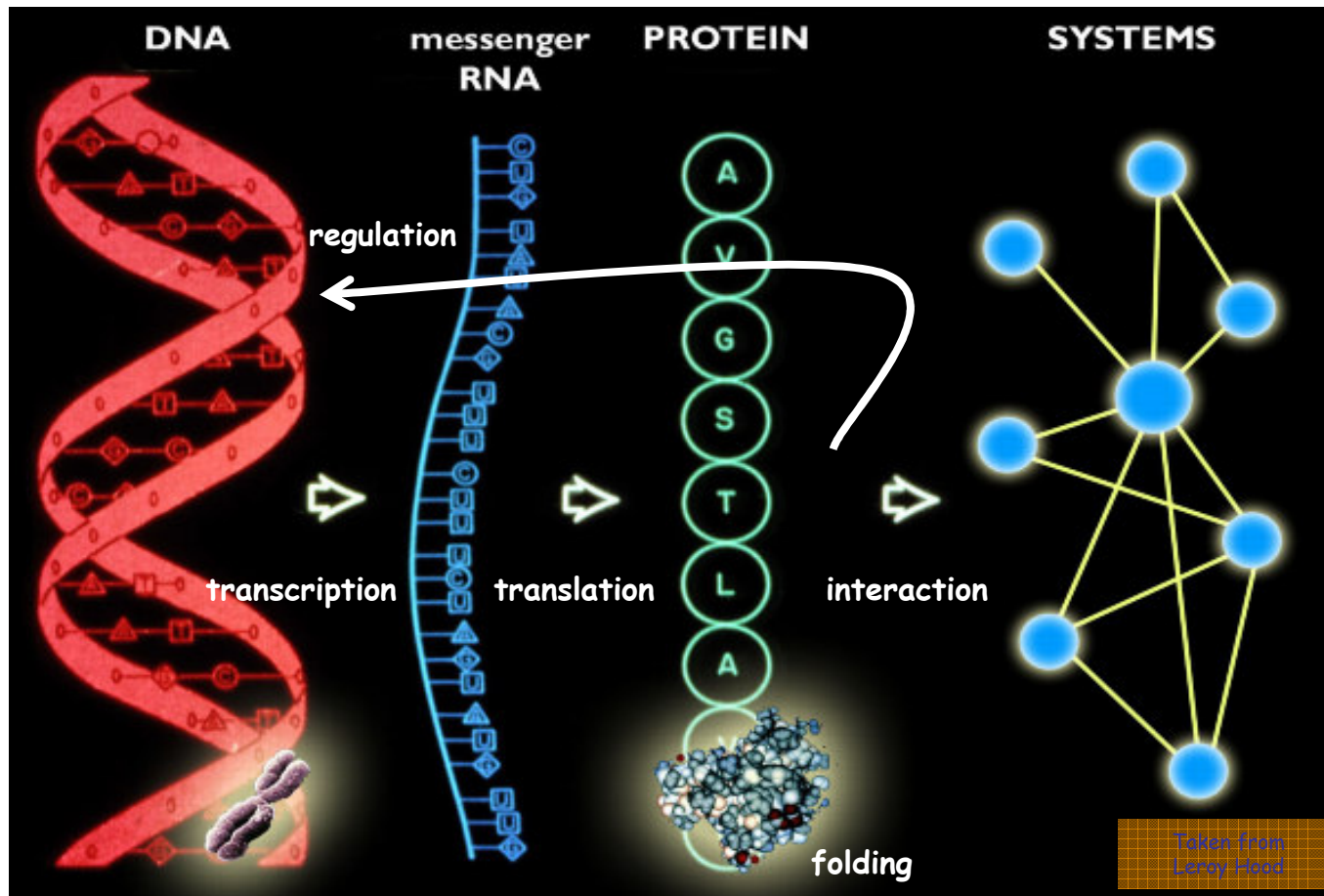
# Notations for the Protein Machine

- Stochastic  $\pi$ -Calculus
  - Priami (following Hillston's PEPA) formalizes a stochastic version of  $\pi$ -calculus where channels have communication *rates*.
- BioSPi
  - Regev-Shapiro-Silverman propose modeling chemical interactions (exchange of electrons and small molecules) as "communication".
  - Standard stochastic simulation algorithms (Gillespie) can be used to run in-silico experiments.
  - Complex formation is encoded via  $\pi$ -restriction.
- PEPA
  - Calder Gilmore and Hillston model the ERK pathway.
- k-calculus
  - Danos and Laneve (following Kitano's BioCalculus) define a calculus where complex formation is primitive.
- (Stochastic) Petri Nets
  - S.Reddy'94 modeling pathways.
  - Srivastava Peterson and Bentley analyze and simulate E.coli stress response circuit.
- Bio State Charts
  - Harel uses State Charts to model biological interactions via a semi-graphical FSM notation.
- Pathway Logic
  - Talcott-Eker-Knapp-Lincoln use term-rewriting.
- BioCham
  - ChabrierRivier-Fages-Soliman use term-rewriting and CLT modelchecking.
- Kohn Diagrams, Kitano Diagrams
- SBML (Systems Biology Markup Language)
  - XML dialect for MIM's:
    - Compartments (statically nested)
    - Reagents with concentrations
    - Reactions with various rate laws
  - Read and written by many tools via the Systems Biology Workbench protocol
    - Graph editors
    - Simulators (including simulation web services)
    - Databases

# 2. The Gene Machine

*Pretty far from the atoms.*

The "Central Dogma" of Molecular Biology



4-letter digital code

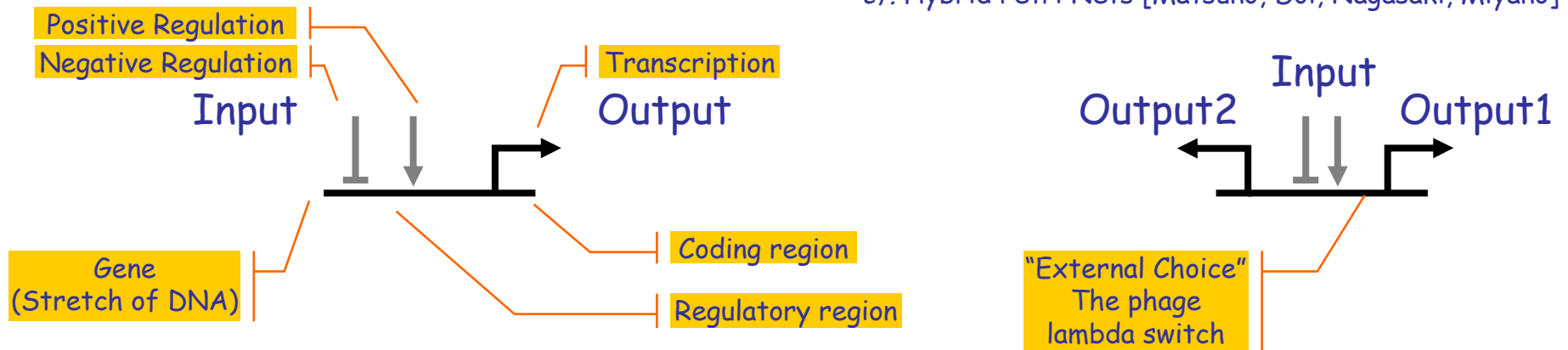
4-letter digital code

20-letter digital code

50.000(?) shapes

# The Gene Machine "Instruction Set"

cf. Hybrid Petri Nets [Matsuno, Doi, Nagasaki, Miyano]



Regulation of a gene (positive and negative) influences transcription. The regulatory region has precise DNA sequences, but not meant for coding proteins: meant for binding regulators.

Transcription produces molecules (RNA or, through RNA, proteins) that bind to regulatory region of other genes (or that are end-products).

## Human (and mammalian) Genome Size

3Gbp (Giga base pairs) 750MB @ 4bp/Byte (CD)

Non-repetitive: 1Gbp 250MB

In genes: 320Mbp 80MB

Coding: 160Mbp 40MB

Protein-coding genes: 30,000-40,000

## M.Genitalium (smallest true organism)

580,073bp 145KB (eBook)

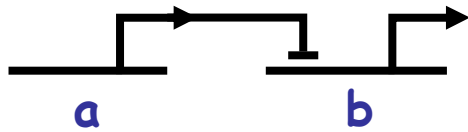
E.Coli (bacteria): 4Mbp 1MB (floppy)

Yeast (eukarya): 12Mbp 3MB (MP3 song)

Wheat 17Gbp 4.25GB (DVD)



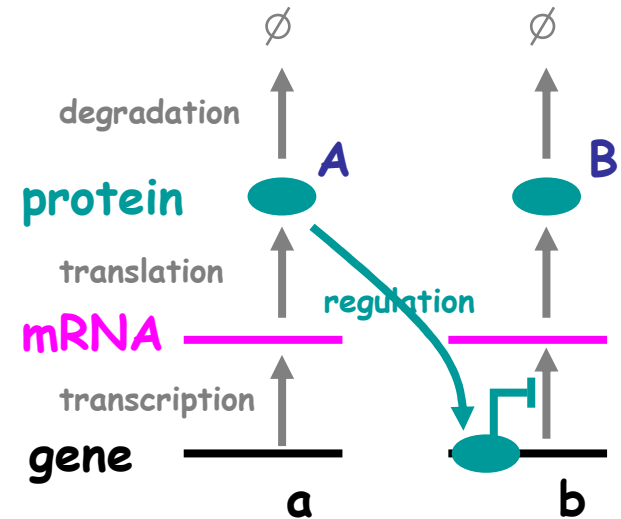
# Gene Composition



Is a shorthand for:

Under the assumptions [Kim & Tidor]

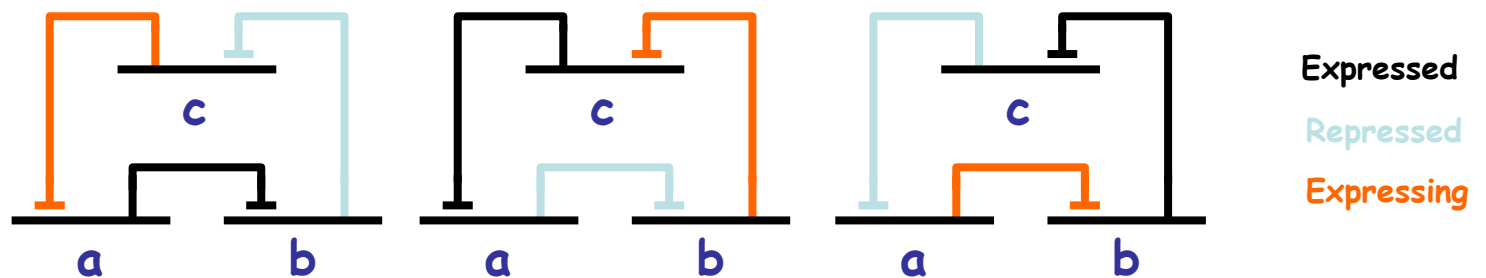
- 1) The solution is well-stirred  
(no spatial dependence on concentrations or rates).
- 2) There is no regulation cross-talk.
- 3) Control of expression is at transcription level only  
(no RNA-RNA or RNA-protein effects)
- 4) Transcriptions and translation rates monotonically affect mRNA and protein concentrations resp.



Ex: Bistable Switch



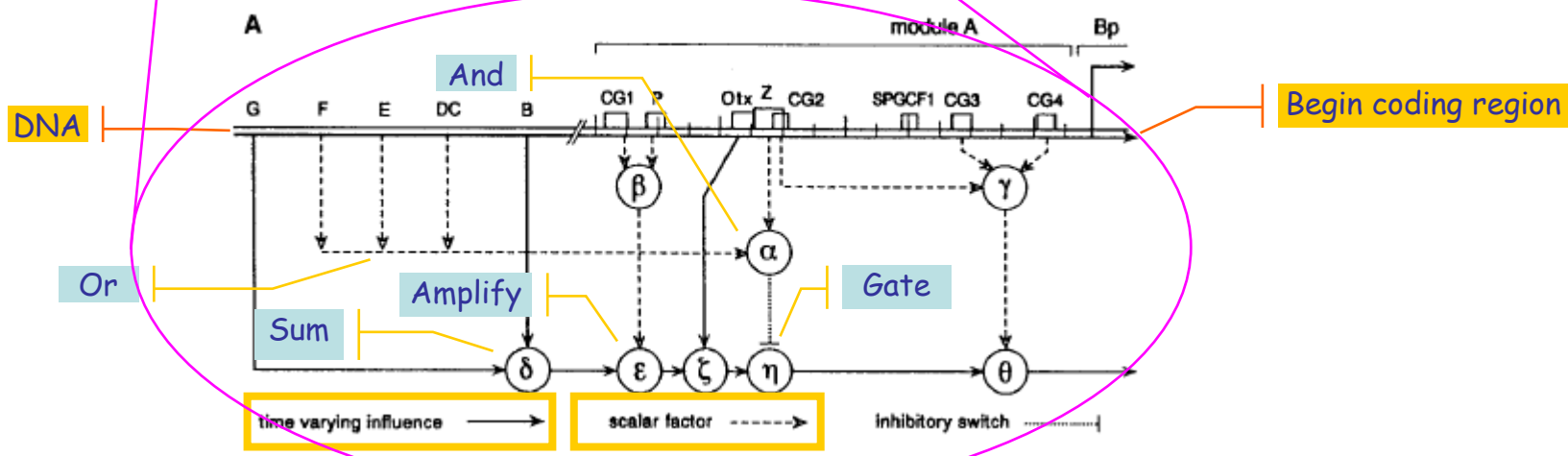
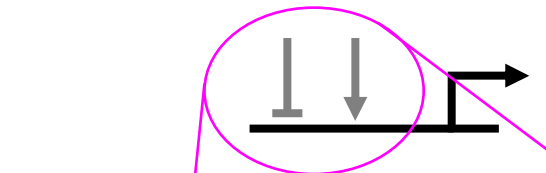
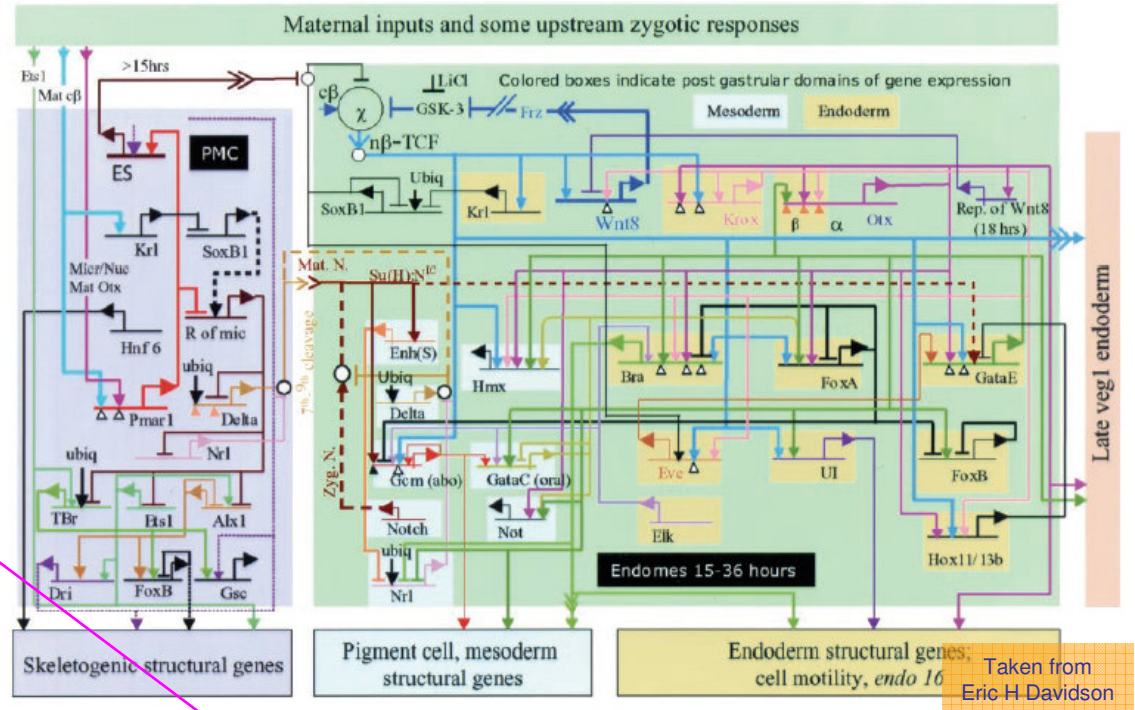
Ex: Oscillator



# Gene Regulatory Networks

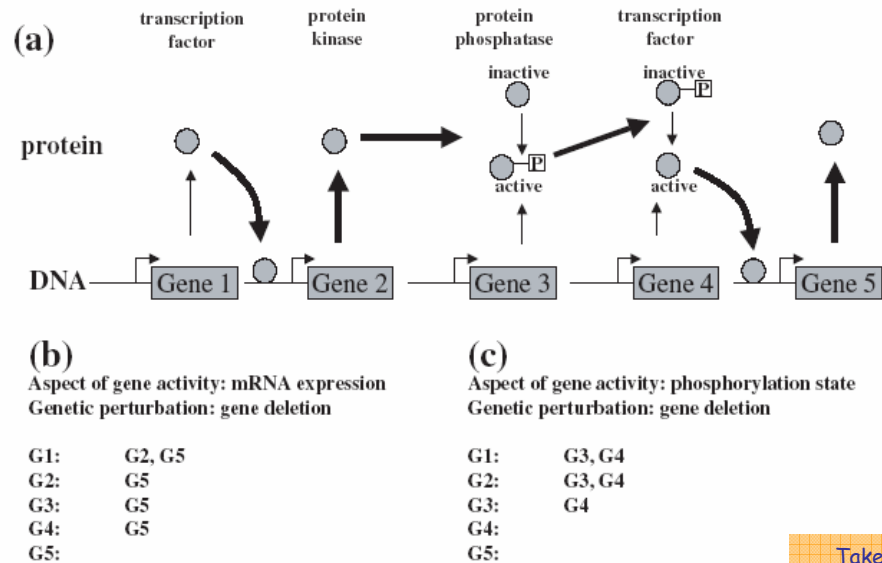
<http://strc.herts.ac.uk/bio/maria/NetBuilder/>

NetBuilder



# Indirect Gene Effects

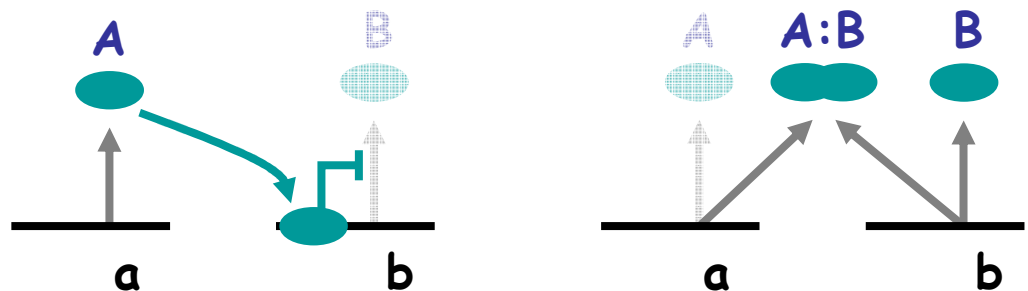
No combination of standard high-throughput experiments can reconstruct an a-priori known gene/protein network [Wagner].



Taken from Andreas Wagner

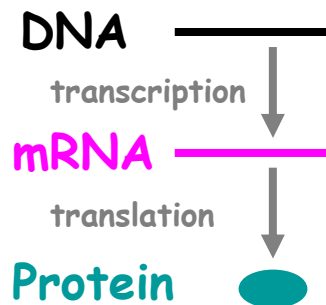
Fig. 1. The importance of specifying gene activity when reconstructing genetic networks. (a) A hypothetical biochemical pathway involving two transcription factors, a protein kinase, and a protein phosphatase, as well as the genes encoding them. See text for details. (b) Shown is a list of perturbation effects for each of the five genes in (a), when perturbing individual genes by deleting them, and when using mRNA expression level as an indicator of gene activity. The left-most symbol in each line stands for the perturbed gene. To the right of each colon is a list of genes whose activity is affected by the perturbation. (c) Analogous to (b) but for a different notion of gene activity (phosphorylation state).

One of many bistable switches that cannot be described by pure gene regulatory networks [Francois & Hakim].



# Structure of the Coding Region

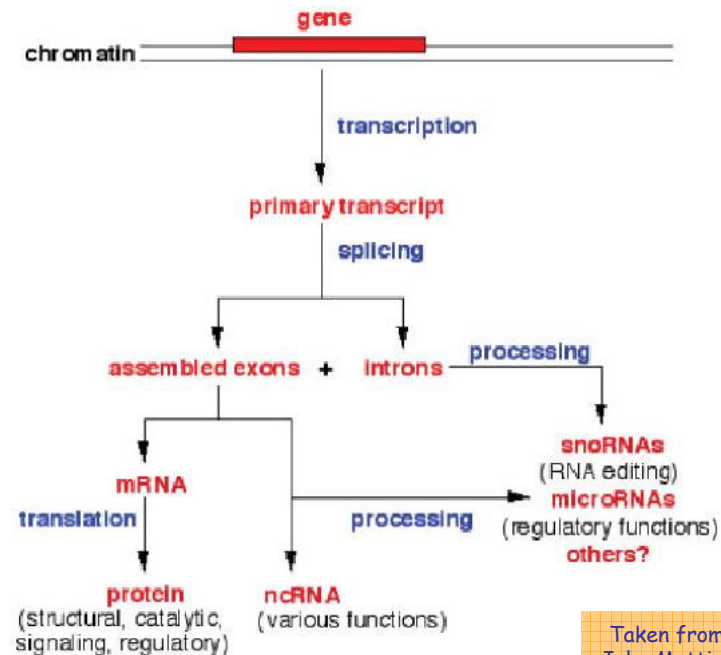
## The Central Dogma



RNA is not just an intermediary; it can:

- Fold-up like a protein
- Act like an enzyme
- Regulate other transcribed RNA
- Direct protein editing
- ...

## Challenging the Dogma (in higher organisms)



Taken from John Mattick

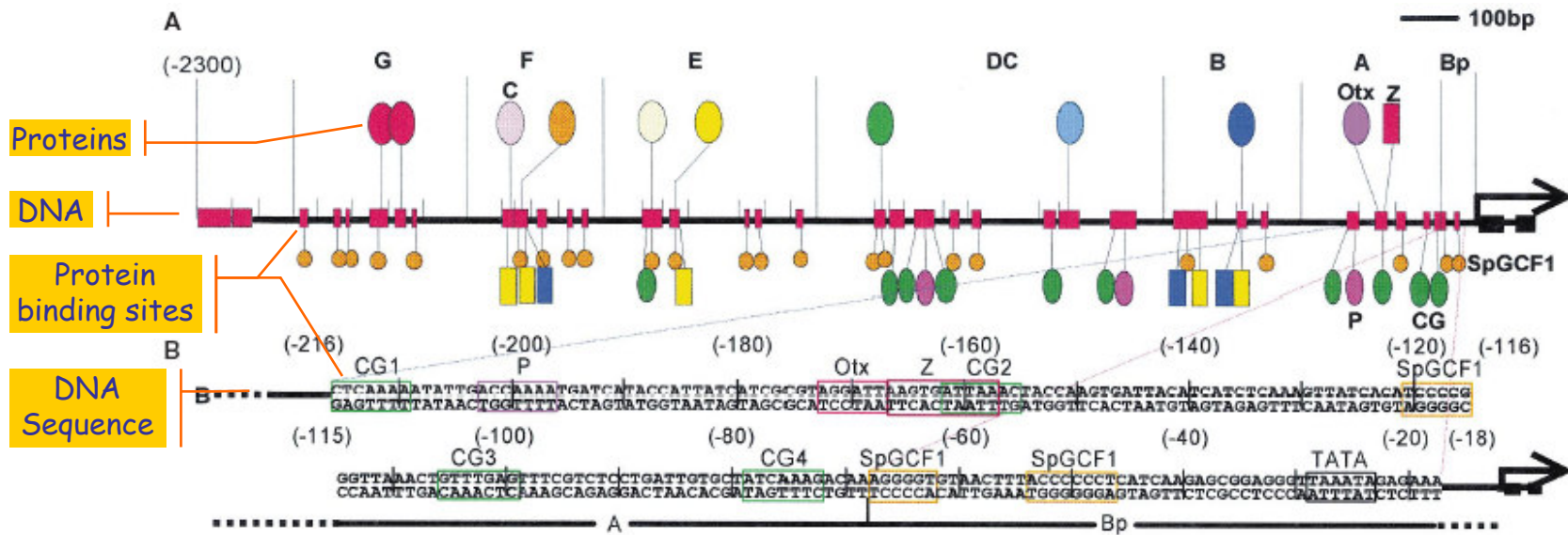
97-98% of the transcriptional output of the human genome is non-protein-coding RNA.

30-40,000 "protein genes" (1.5% of genome)

60-100,000 "transcription units" (>30% of genome is transcribed)



# Structure of a Regulatory Region



2300bp!  
> average protein

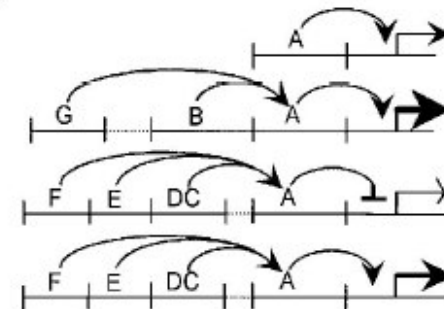
## C Module A functions:

Vegetal plate expression in early development:

Synergism with modules B and G enhancing endoderm expression in later development:

Repression in ectoderm (modules E and F) and skeletogenic mesenchyme (module DC):

Modules E, F and DC with LiCl treatment:



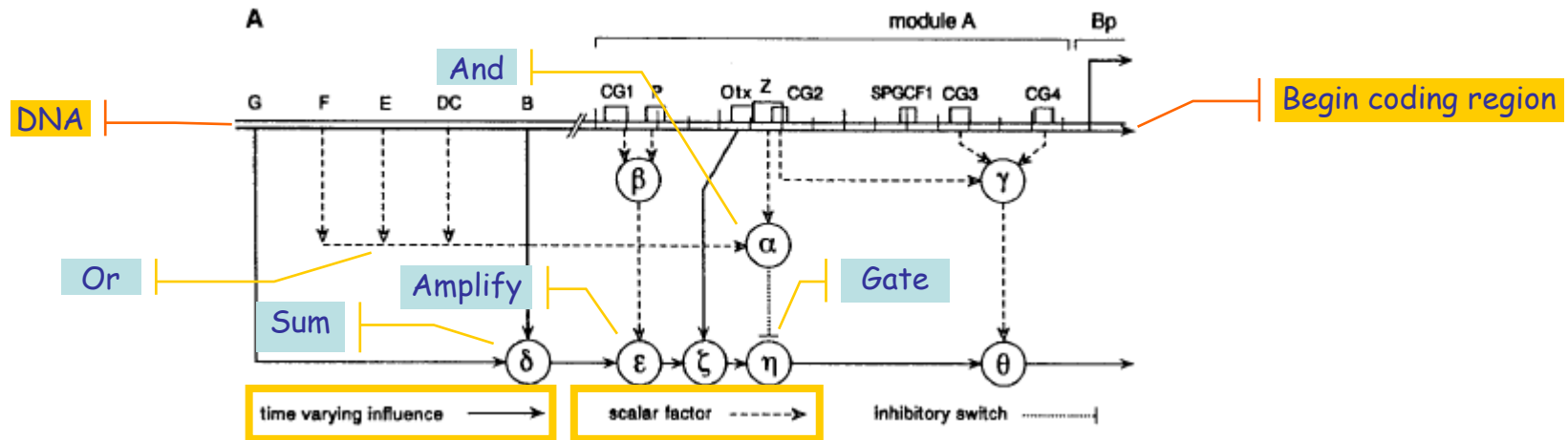
**Fig. 1.** *Endo16* cis-regulatory system and interactive roles of module A. (A) Diversity of protein binding sites and organization into modular subregions [modified from (7)]. Specific DNA binding sites are indicated as red blocks; modular subregions are denoted by letters G to A (Bp, basal promoter). Proteins binding at the target sites considered in this work are indicated: Otx, SpOtx-1 (12); SpGCF1 (14); the proteins CG, Z, and P, which are not yet cloned; and protein C [a CREB family protein (18)] in subregion F. Proteins for which sites occur in multiple regions of the DNA sequence (indicated by the black line) are shown beneath. (B) Sequence of module A and location of protein binding sites. Sites are indicated in the same colors as in (A). A fragment containing CG<sub>3</sub> and CG<sub>4</sub> sites as well as Bp has no endoderm-

specific activity and services other upstream cis-regulatory systems promiscuously; similarly, the *Endo16* cis-regulatory system functions specifically with heterologous promoters substituted for Bp (5, 8, 19). Boxed sequences indicate conserved core elements of the target sites (7, 12, 14), not the complete target site sequences. (C) Integrative and interactive functions of module A (5, 8). Module A communicates the output of all upstream modules to the basal transcription apparatus. It also initiates endoderm expression, increases the output of modules B and G, and is required for functions of the upstream modules F, E, and DC. These functions are repression of expression in nonendodermal domains and enhancement of expression in response to LiCl.





# Function of a Regulatory Region



B

if (F = 1 or E = 1 or CD = 1) and (Z = 1)      Repression functions of modules F, E, and DC mediated by Z site  
 $\alpha = 1$

else  $\alpha = 0$

if (P = 1 and CG<sub>1</sub> = 1)      Both P and CG<sub>1</sub>, needed for synergistic link with module B  
 $\beta = 2$

else  $\beta = 0$

if (CG<sub>2</sub> = 1 and CG<sub>3</sub> = 1 and CG<sub>4</sub> = 1)      Final step up of system output  
 $\gamma = 2$

else  $\gamma = 1$

$\delta(t) = B(t) + G(t)$       Positive input from modules B and G

$\epsilon(t) = \beta * \delta(t)$       Synergistic amplification of module B output by CG<sub>1</sub>-P subsystem

if ( $\epsilon(t) = 0$ )      Switch determining whether Otx site in module A, or upstream modules (i.e., mainly module B), will control level of activity  
 $\xi(t) = Otx(t)$

else  $\xi(t) = \epsilon(t)$

if ( $\alpha = 1$ )      Repression function inoperative in endoderm but blocks activity elsewhere  
 $\eta(t) = 0$

else  $\eta(t) = \xi(t)$

$\theta(t) = \gamma * \eta(t)$       Final output communicated to BTA



# Gene Machine Programs

- All that goes to show that:
  - The faithful description of even a simple genetic network is probably going to require writing a fairly substantial "program"/model.



# The Programming Model

Strange facts about genetic networks:

**Not an operator algebra.** The output of each gate is fixed and pre-determined; it is never a function of the input!

**Not term-rewriting, nor Petri nets.** Inhibition is widespread.

**Not Communicating Sequential Processes.** Feedback is widespread: asynchronous communication needed to avoid immediate self-deadlocks. Even the simplest gates cannot be modeled as a single synchronous process.

**Not Message-Passing between genes.** Messages themselves have behavior (e.g., they stochastically decay and combine), hence messages are processes as well.

**Not Data-Flow.** Any attempt to use data-flow-style modeling seems doomed because of widespread loops that lead to deadlocks.

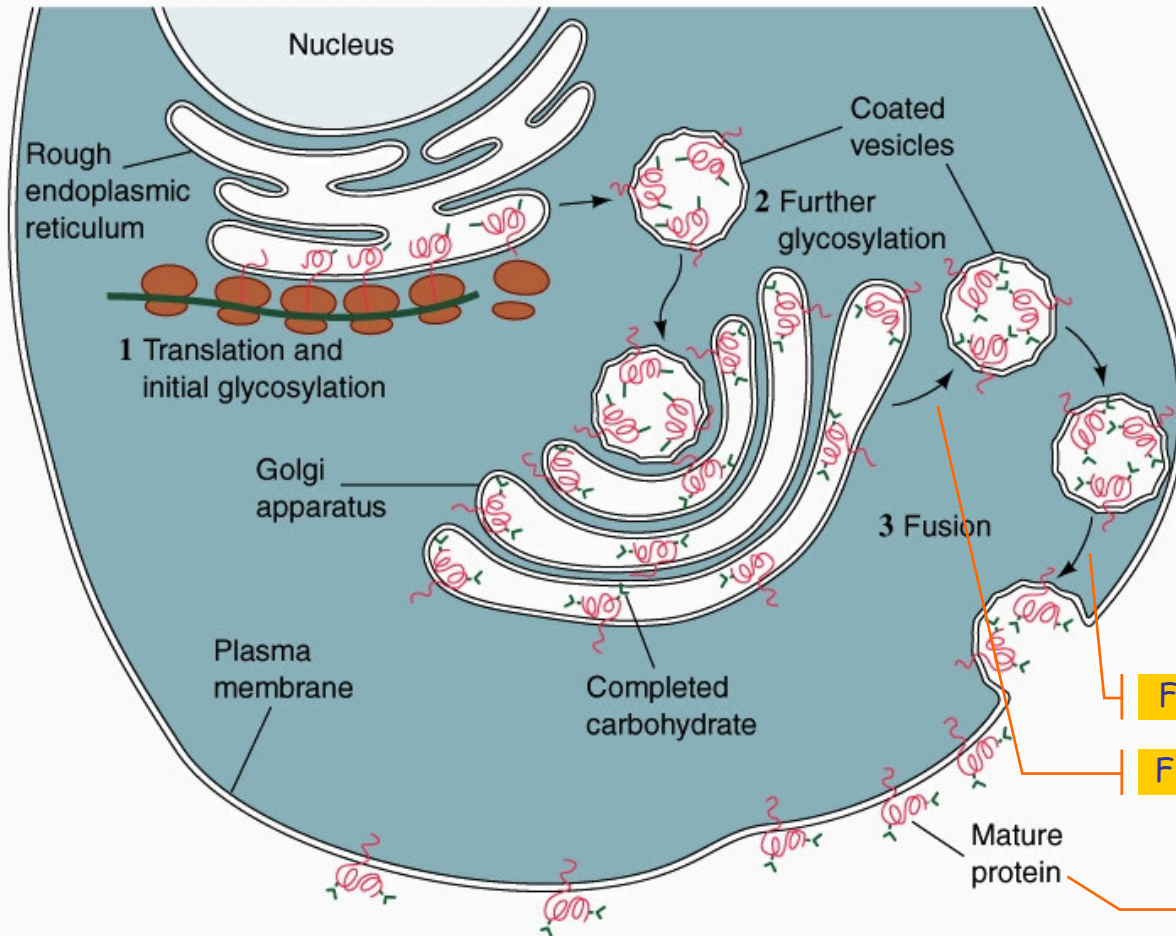
**Stochastic broadcasting.** The apparently crude idea of broadcasting a whole bunch of asynchronous decaying messages to activate a future gate, means there are never any "pipeline full" deadlocks, even in presence of abundant feedback loops.

**Stochastic degradation.** Degradation is fundamental for system stability, and at the same time can lead to sudden instability and detection of concentration levels.

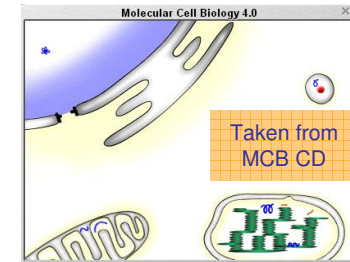
# Notations for the Gene Machine

- Many of the same techniques as for the Protein Machine apply.
  - Process Calculi, Petri Nets, Term-Rewriting Systems...
- But the “programming model” is different.
  - Asynchronous stochastic control.
  - Biologically poorly understood.
  - Network “motifs” are being analyzed.
- Specific techniques:
  - Hybrid Petri Nets
    - [Matsuno, Doi, Nagasaki, Miyano] Gene Regulation
    - Genomic Object Net [www.genomicobject.net](http://www.genomicobject.net)
  - Gene Regulation Diagrams
  - Mixed Gene-Protein Diagrams

# 3. The Membrane Machine *Very far from the atoms.*



Molecular transport and transformation through dynamic compartment **fusion and fission**.



Fusion

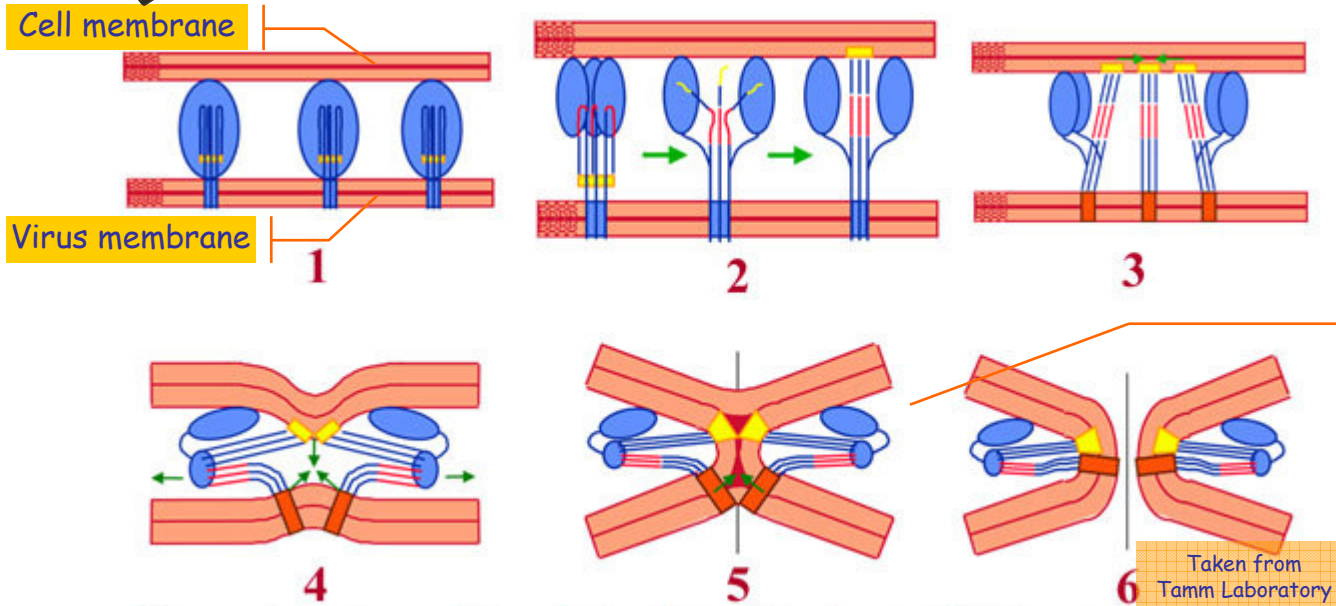
Fission

} The Instruction Set

Well, what is all that for?  
"Given the complicated pathways that have evolved to synthesize them, it seems likely that these [modified proteins] have important functions, but for the most part these functions **are not known**" [MBP p.609]

# Membrane Fusion

Positive curvature to Negative curvature transition in 3D

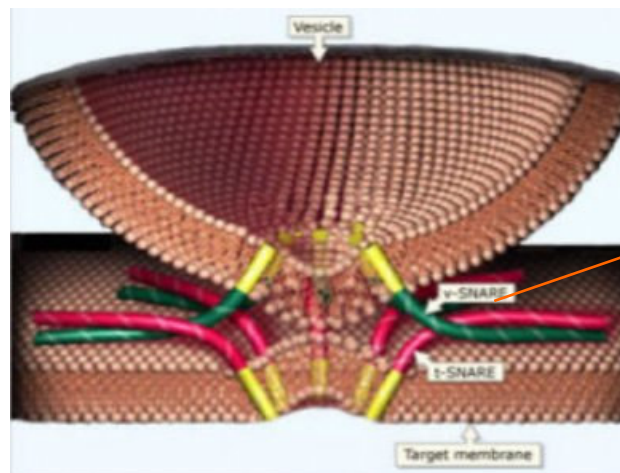


Proposed sequence of events in pH sensitive hemagglutinin membrane fusion

**Aggressive fusion (virus)**

By unknown mechanisms, the exoplasmic leaflets of the two membranes fuse" [MCB p745]

**Cooperative fusion (vesicle)**

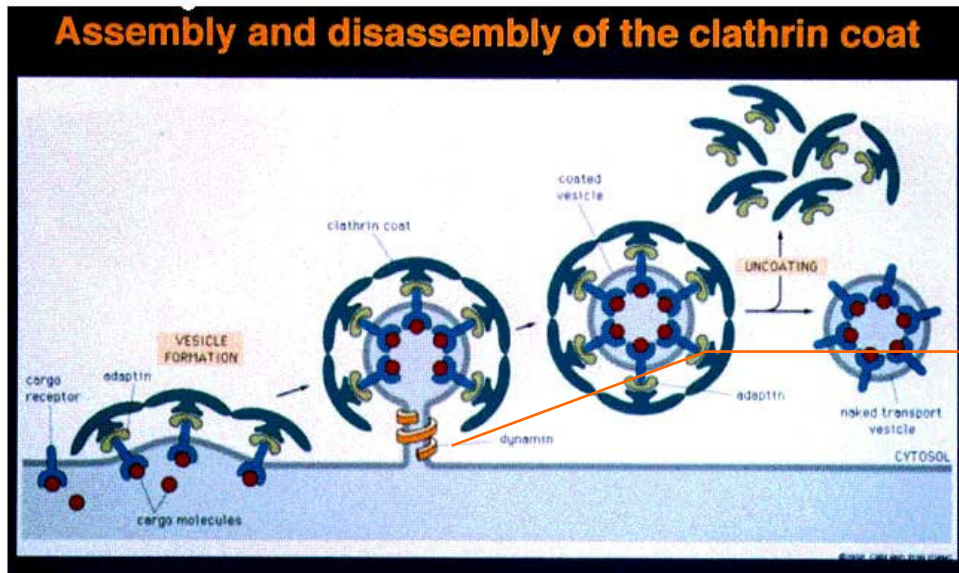


"Fusion of the two membranes immediately follows prefusion, but precisely how this occurs is not known" [MCB p742]



# Membrane Fission

Negative curvature to Positive curvature transition in 3D

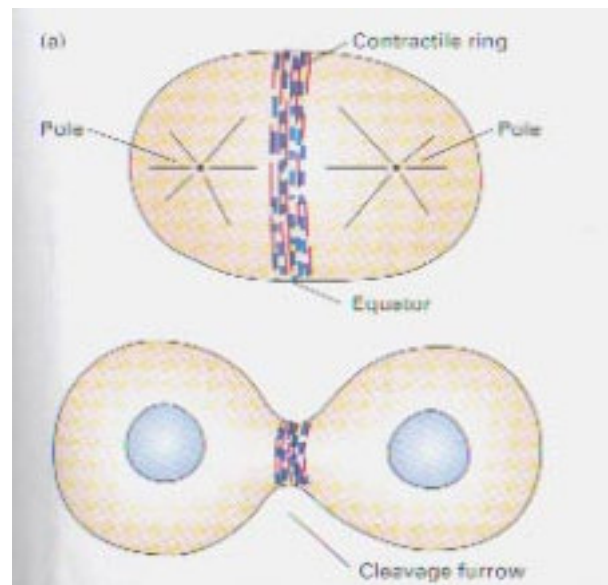


## Vesicle Formation



Movie by Allison Bruce

"Nonetheless, the actual process whereby a segment of phospholipid bilayer is 'pinched off' to form a pit and eventually a new vesicle is still not understood" [MCB p.746]

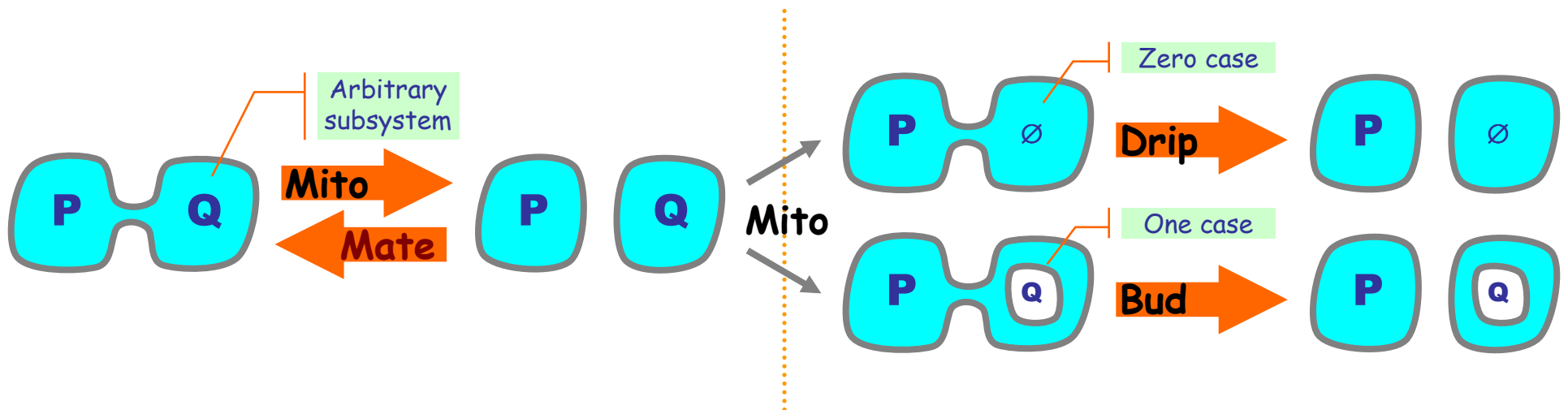
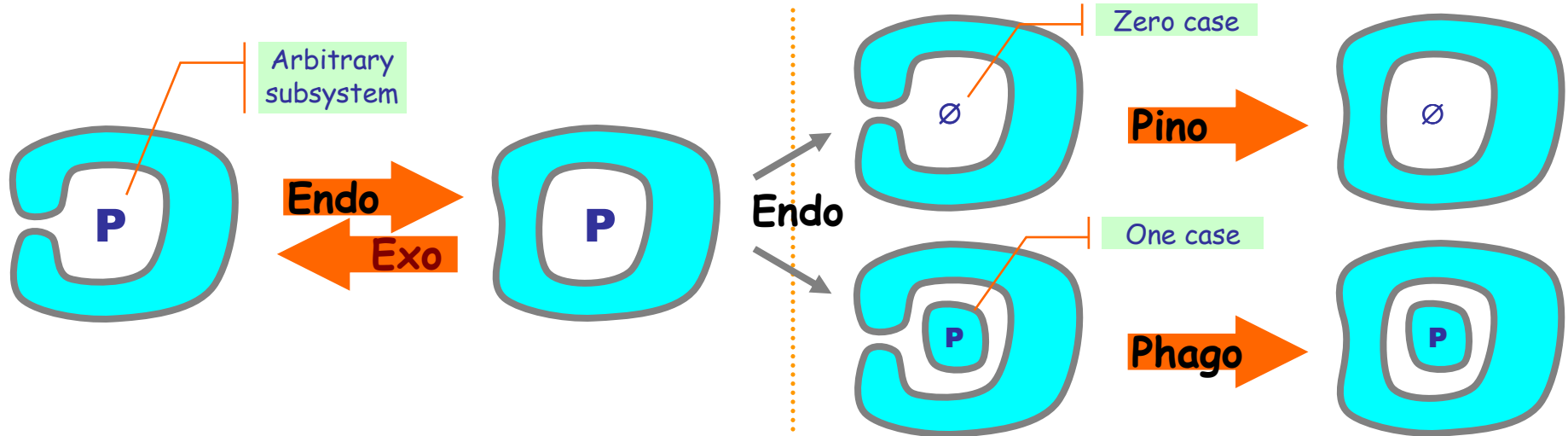


## Cytokinesis (Mitosis)

# Notations for the Membrane Machine

- “Snapshot” diagrams
  - In biology literature.
- P-Systems
  - G.Paun uses ideas from the theory of grammars and formal languages to model “Membrane Computing” (book 2002).  
<http://psystems.disco.unimib.it/>.
- BioAmbients
  - An extension of BioSPI along Ambient Calculus lines (with more bio-relevant mobility primitives) to model dynamic compartments.
- Brane Calculi
  - Computation *on* the membrane...

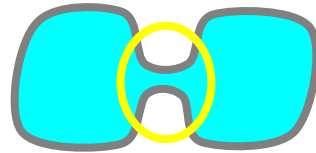
# The Membrane Machine "Instruction Set"



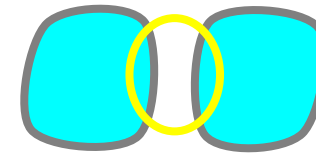


# Locally Implementable!

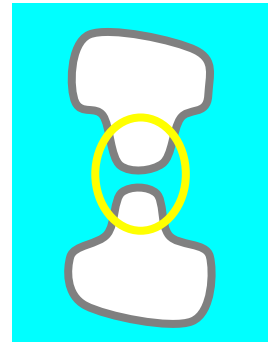
Global Views



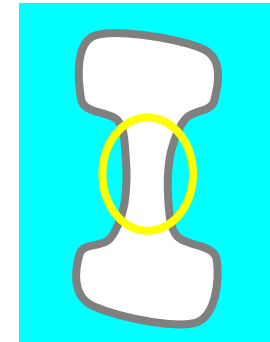
Mito →



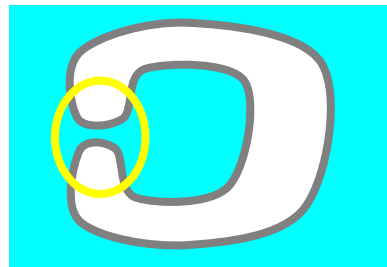
(Fission)



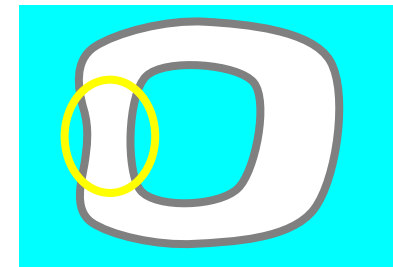
Mate →



(Fusion)



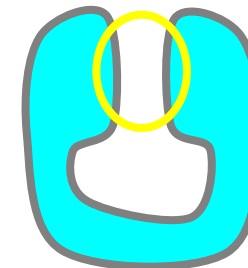
Endo →



(Fission)



Exo →

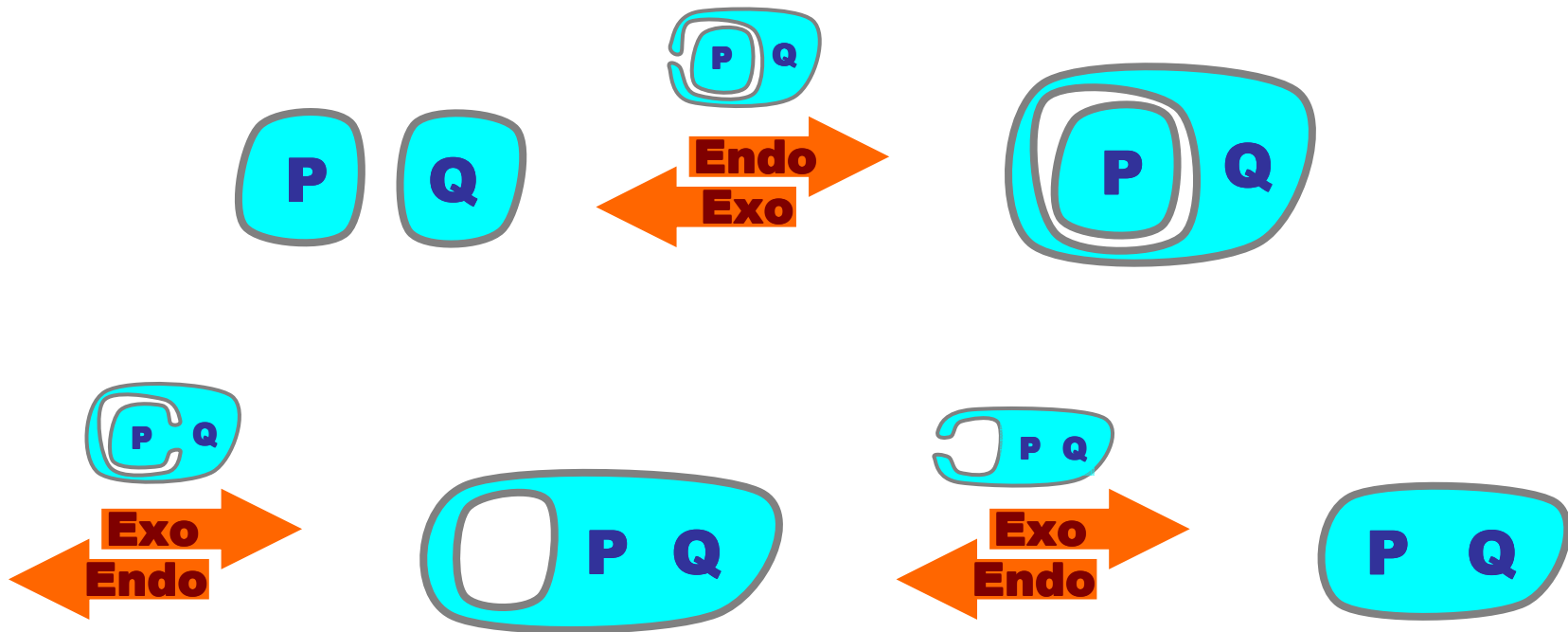


(Fusion)

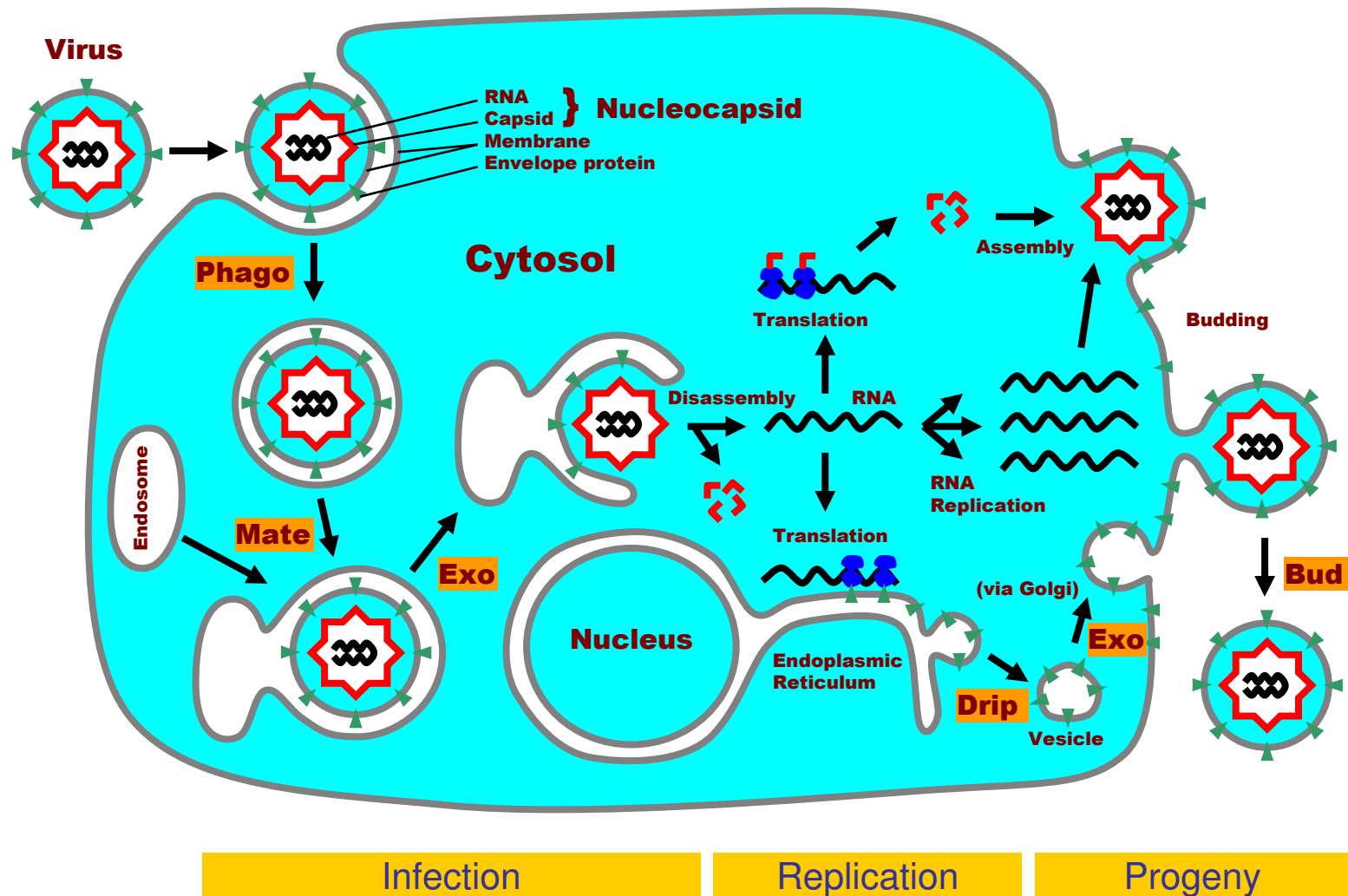


Same  
Local  
View!

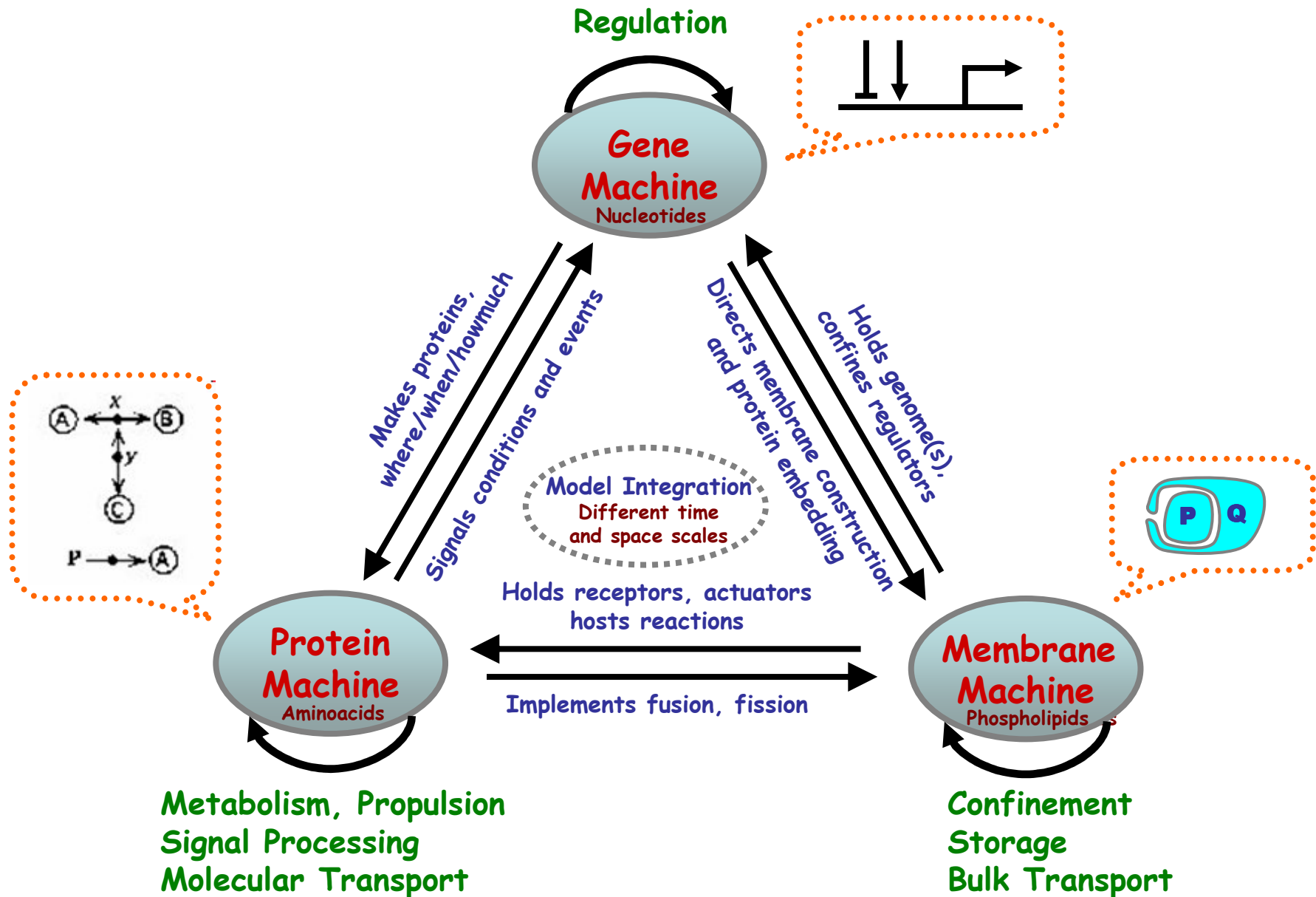
# Mito/Mate by 3 Endo/Exo



# Ex: Viral Reproduction



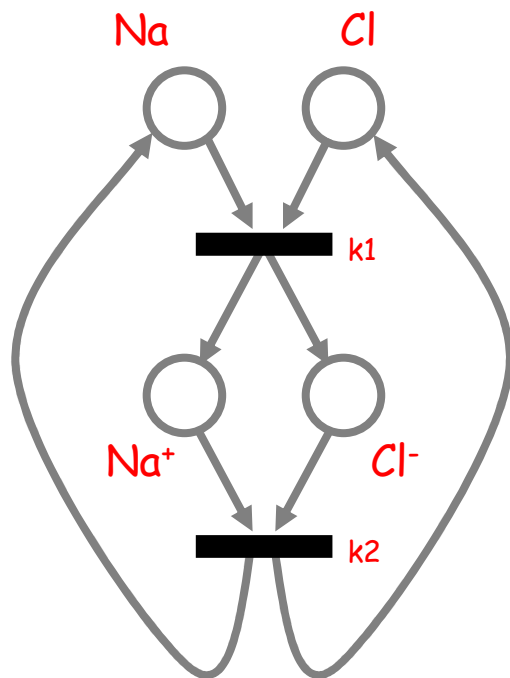
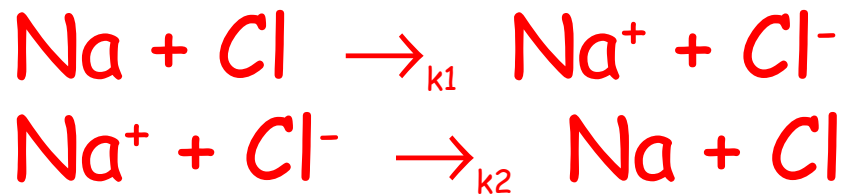
# Abstract Machines of Biochemistry



# Process Calculi

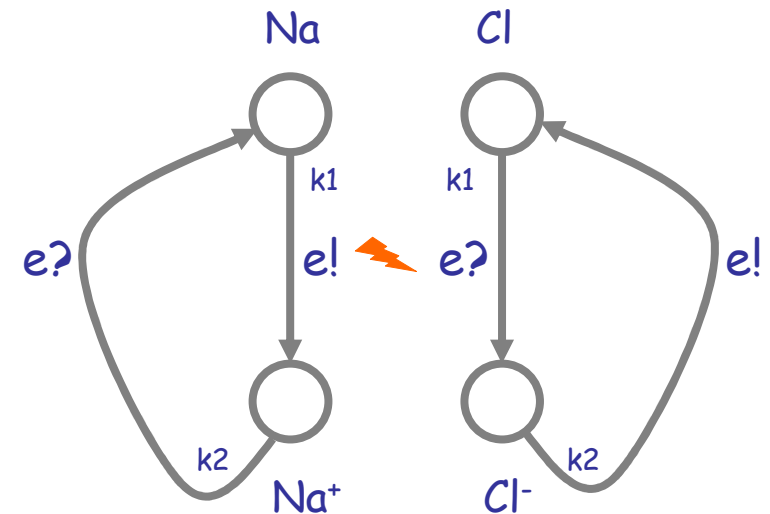
# Chemistry vs. $\pi$ -calculus

A process calculus (chemistry, or SBML)



(Can be converted to a CTMC)

The same "model"



(Can be converted to a CTMC)

$$\text{Na} = e_{k_1}!. \underbrace{e_{k_2}?. \text{Na}}_{\text{Cl}^-}$$

$$\text{Cl} = e_{k_1}?. \underbrace{e_{k_2}!. \text{Cl}}_{\text{Na}^+}$$

A different process calculus

This graphical representation degenerates into spaghetti diagrams: precise and dynamic, but not scalable, structured, or maintainable.



# Stochastic $\pi$ -calculus Executive Summary

- A process calculus:
  - The modular representation of concurrent (and stochastic) processes of all kinds.
  - Cuts down to CTMCs in the finite case (not always), then standard tools are applicable.
  - Can be given friendly automata-like scalable graphical syntax (work in progress).
  - Is directly executable (e.g. via Gillespie).
  - Is analyzable (large body of literature, at least in the non-stochastic case).

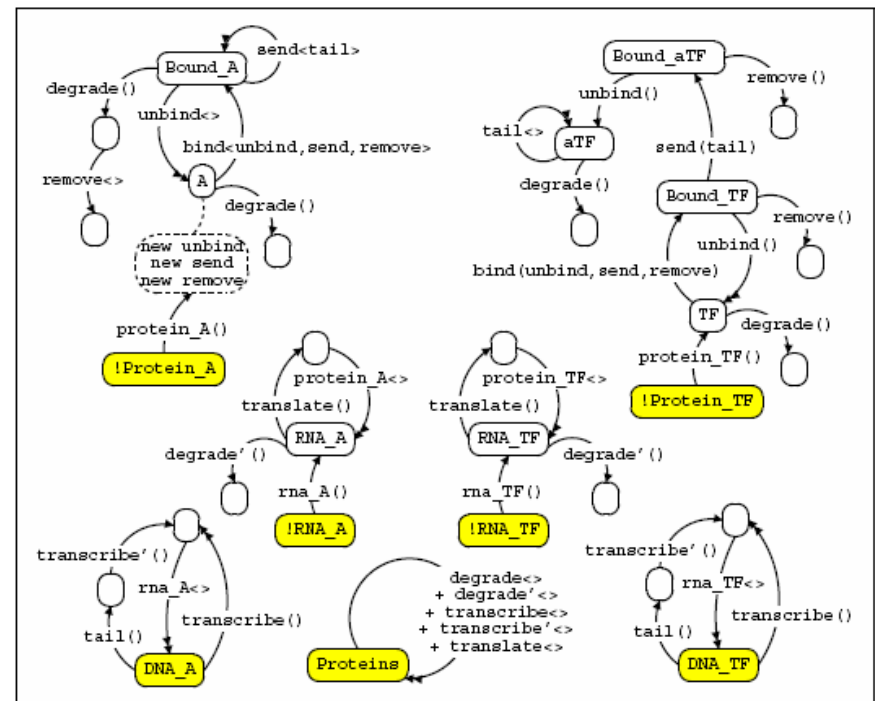


Figure 2. Regulating Gene Expression by Positive Feedback [9]

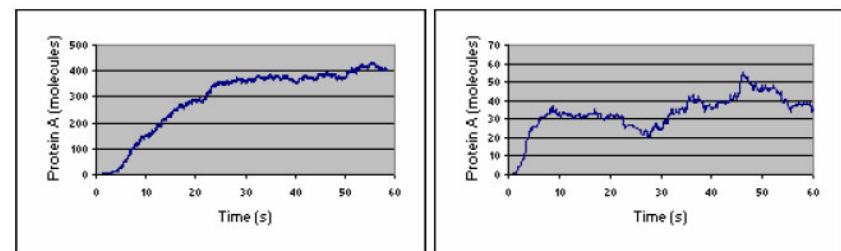


Figure 3. Protein A molecules v.s. time in presence (left) and absence (right) of TF

# Regev-Shapiro: "Molecules as Computation"

Molecule	Process
Interaction capability	Channel
Interaction	Communication
Modification	State change

Cellular Abstractions: Cells as Computation  
Regev&Shapiro NATURE vol 419, 2002-09-26, 343

This mapping works well both for the "protein machine" (synchronous communication) and the "gene machine" (asynchronous communication). But is not enough for the "membrane machine".

# $\pi$ -calculus

## Syntax

$$\pi ::= x(y) \text{ receive } y \text{ along } x \\ \bar{x}(y) \text{ send } y \text{ along } x$$

$$P ::= 0 \mid \sum_{i \in I} \pi_i.P_i \mid [x = y] P \mid P_1 \mid P_2 \mid (\text{new } x)P \mid !P$$

## Structural congruence

### Renaming of bound variables

$$\begin{aligned} x(y).P &= x(z).(\{z/y\}P) && \text{if } z \notin FN(P) \\ (\text{new } y).P &= (\text{new } z).(\{z/y\}P) && \text{if } z \notin FN(P) \end{aligned}$$

## Structural congruence laws

$P \mid Q \equiv Q \mid P$	commutativity of parallel composition
$(P \mid Q) \mid R \equiv P \mid (Q \mid R)$	associativity of parallel composition
$P + Q \equiv Q + P$	commutativity of summation
$(P + Q) + R \equiv P + (Q + R)$	associativity of summation
$(\text{new } x)0 \equiv 0$	restriction of inert processes
$(\text{new } x)(\text{new } y)P \equiv (\text{new } y)(\text{new } x)P$	polyadic restriction
$((\text{new } x)P) \mid Q \equiv (\text{new } x)(P \mid Q)$	scope extrusion
$!P \equiv P \mid P$	replication

## Reaction rules

$$(\dots + \bar{x}(z).Q) \mid (\dots + x(y).P) \rightarrow Q \mid P \{z/y\} \quad \text{communication (COMM)}$$

$$\frac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q} \quad \text{reaction under parallel composition (PAR)}$$

$$\frac{P \rightarrow P'}{(\text{new } x)P \rightarrow (\text{new } x)P'} \quad \text{reaction under restriction (RES)}$$

$$\frac{Q \equiv P \quad P \rightarrow P' \quad P' \equiv Q'}{Q \rightarrow Q'} \quad \text{structural congruence (STRUCT)}$$

Syntax

Chemical  
Mixing

Reactions

# Stochastic $\pi$ -calculus

- Stochastic extension of  $\pi$ -calculus. [C.Priami]

Associate a single parameter  $r$  (**rate**) in  $(0, \text{infinity}]$  of an **exponential distribution** to each activity  $a$ ; it describes the stochastic behavior of the activity

$a.P$  is replaced by  $(a, r).P$

Exponential distribution guarantees the **memoryless property**: the time at which a change of state occurs is independent of the time at which the last change of state occurred.

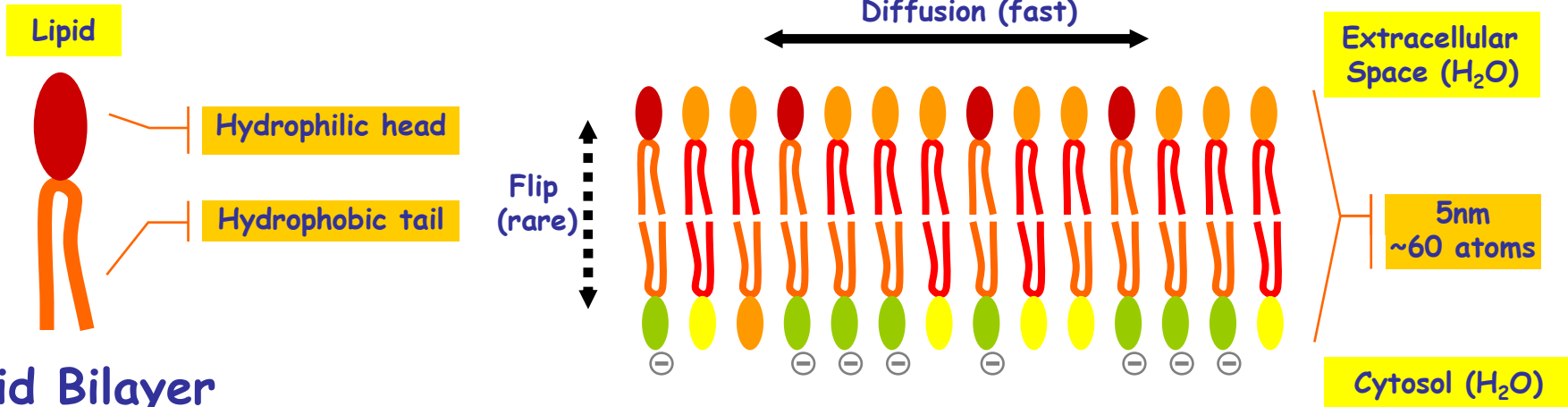
**Race condition** is defined in a **probabilistic competitive** context: all the activities that are enabled in a state compete and the fastest one succeeds.

- New implementation: SPiM. [A.Phillips]. Paper at BioConcur.

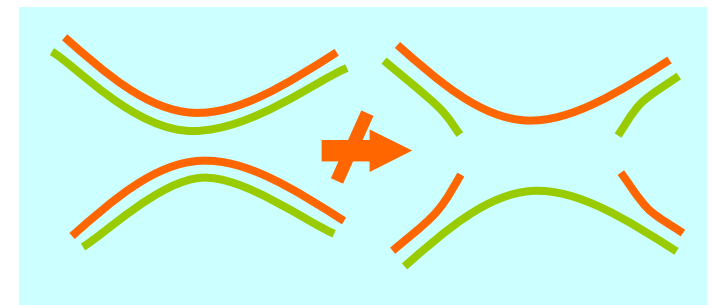
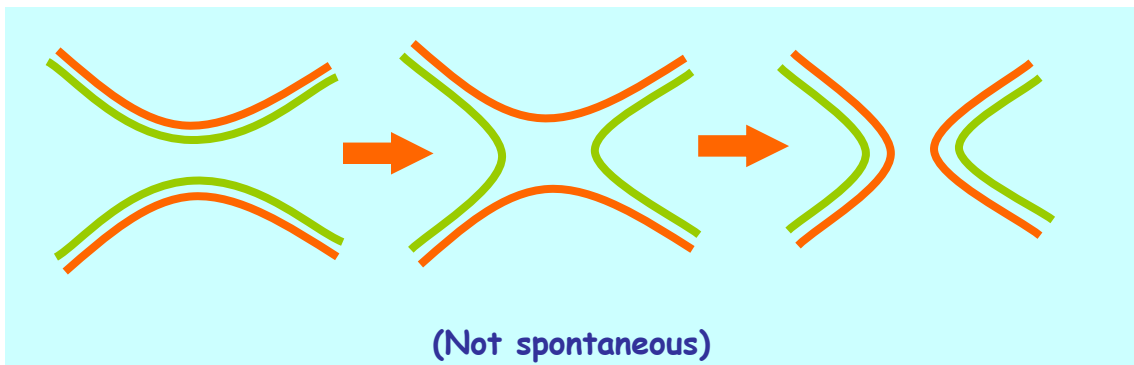
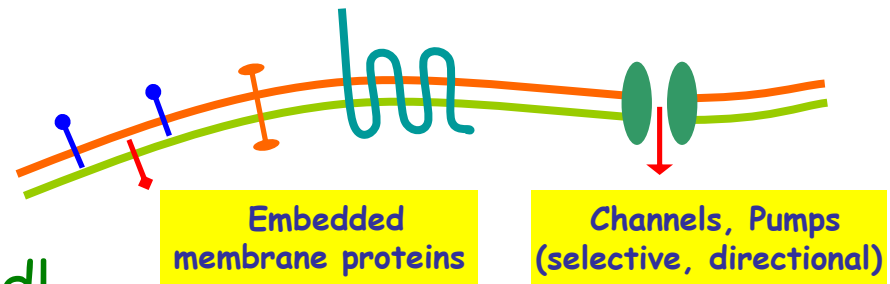
**Brane Calculi**

**Computation "on"  
the membrane**

# Membranes are Oriented 2D Surfaces

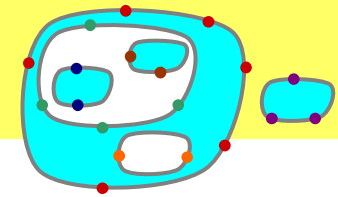


**Lipid Bilayer**  
 Self-assembling  
 Largely impermeable  
 Asymmetrical (in real cells)  
 With embedded proteins  
**A 2D fluid inside a 3D fluid!**





# Brane Calculi



**systems**  $P, Q ::= \diamond \mid P \circ Q \mid !P \mid \sigma(P)$

nests of membranes

**branes**  $\sigma, \tau ::= 0 \mid \sigma \mid \tau \mid !\sigma \mid a.\sigma$

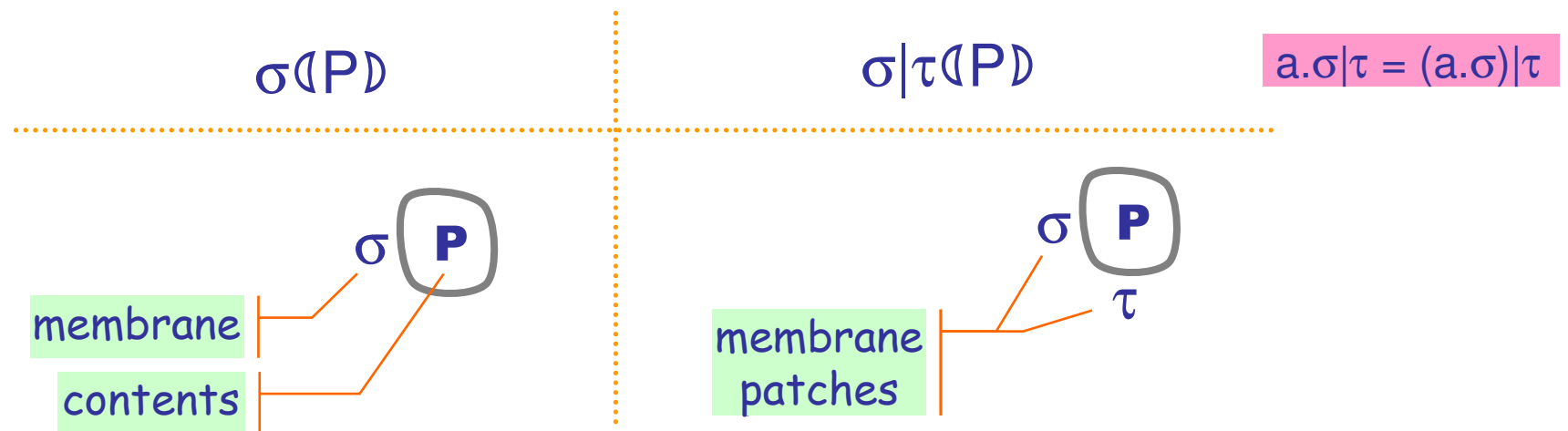
combinations of actions

**actions**  $a ::= 1 \mid \dots$

(fill in as needed)

1D fluids ( $\sigma$ ) inside a 2D fluid ( $P$ )

TWO commutative monoids instead of ONE of normal process calculi



N.B. Restriction ( $\nu n$ ) could be added to both systems and branes. It usually would originate in branes, but would extrude to whole systems.

# Congruence $\equiv$ and Reaction $\rightarrow$

	System	Brane
Fluidity	$P \circ Q \equiv Q \circ P$ $P \circ (Q \circ R) \equiv (P \circ Q) \circ R$ $P \circ \diamond \equiv P$	$\sigma   \tau \equiv \tau   \sigma$ $\sigma   (\tau   \rho) \equiv (\sigma   \tau)   \rho$ $\sigma   0 \equiv \sigma$
Plentitude	$!P \equiv P \circ !P$ etc.	$!\sigma \equiv \sigma   !\sigma$ etc.
Units	$0(\diamond) \equiv \diamond$ Froth/Fizz	$1.\sigma \equiv \sigma$ Inaction
Congruence	$P \equiv Q \Rightarrow P \circ R \equiv Q \circ R$ $P \equiv Q \Rightarrow !P \equiv !Q$ $P \equiv Q \wedge \sigma \equiv \tau \Rightarrow \sigma(P) \equiv \tau(Q)$	$\sigma \equiv \tau \Rightarrow \sigma   \rho \equiv \tau   \rho$ $\sigma \equiv \tau \Rightarrow !\sigma \equiv !\tau$ $\sigma \equiv \tau \Rightarrow a.\sigma \equiv a.\tau$

Reaction is up to congruence  $P \equiv P' \wedge P' \rightarrow Q' \wedge Q' \equiv Q \Rightarrow P \rightarrow Q$

Reactions in solution  $P \rightarrow Q \Rightarrow P \circ R \rightarrow Q \circ R$   
 $P \rightarrow Q \Rightarrow \sigma(P) \rightarrow \sigma(Q)$

This is the whole semantics, except for the effects of individual actions.

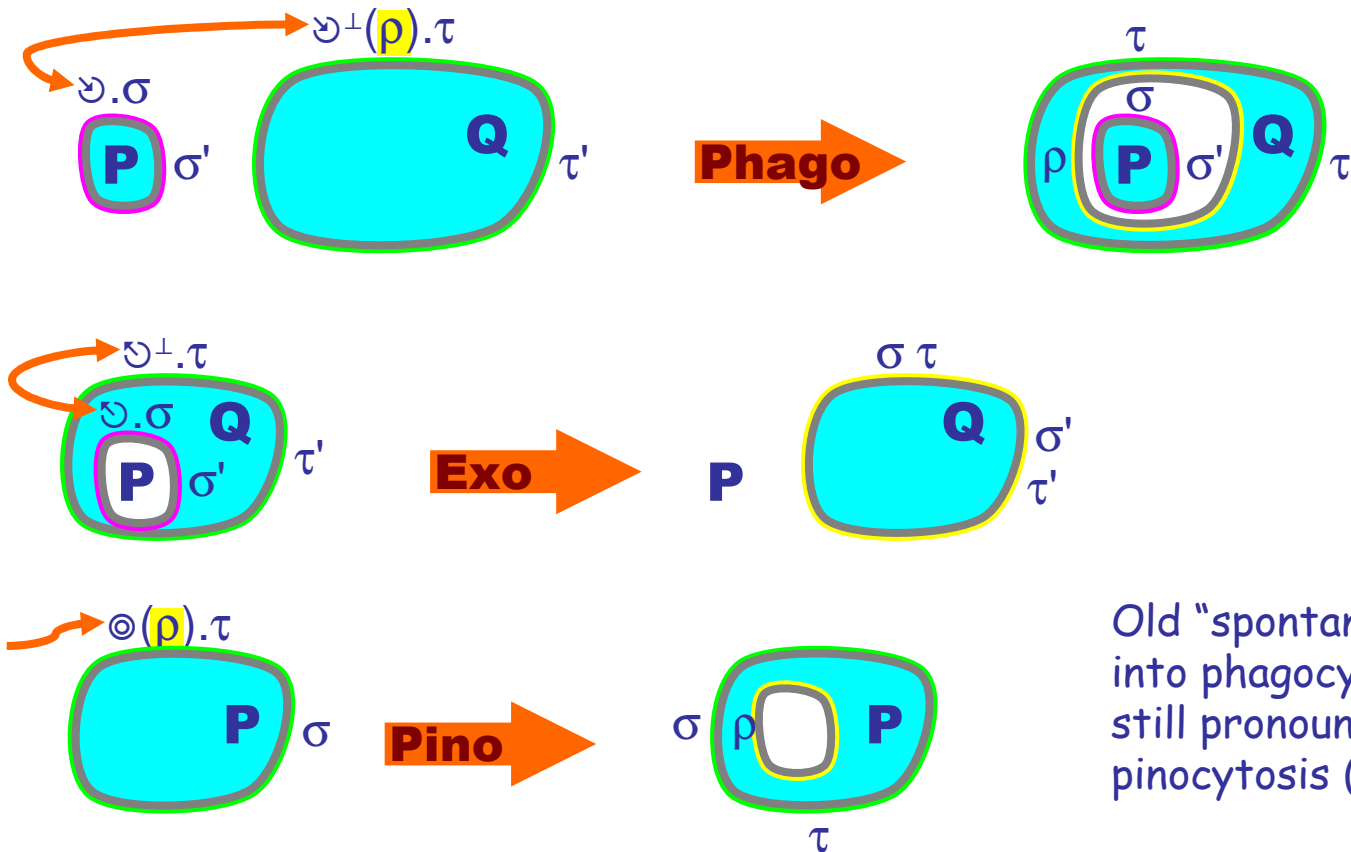
# Brane Reactions

actions

$a ::= \dots \mid \vartheta_n \mid \vartheta_n^\perp(\rho) \mid \vartheta_n \mid \vartheta_n^\perp \mid \odot(\rho)$

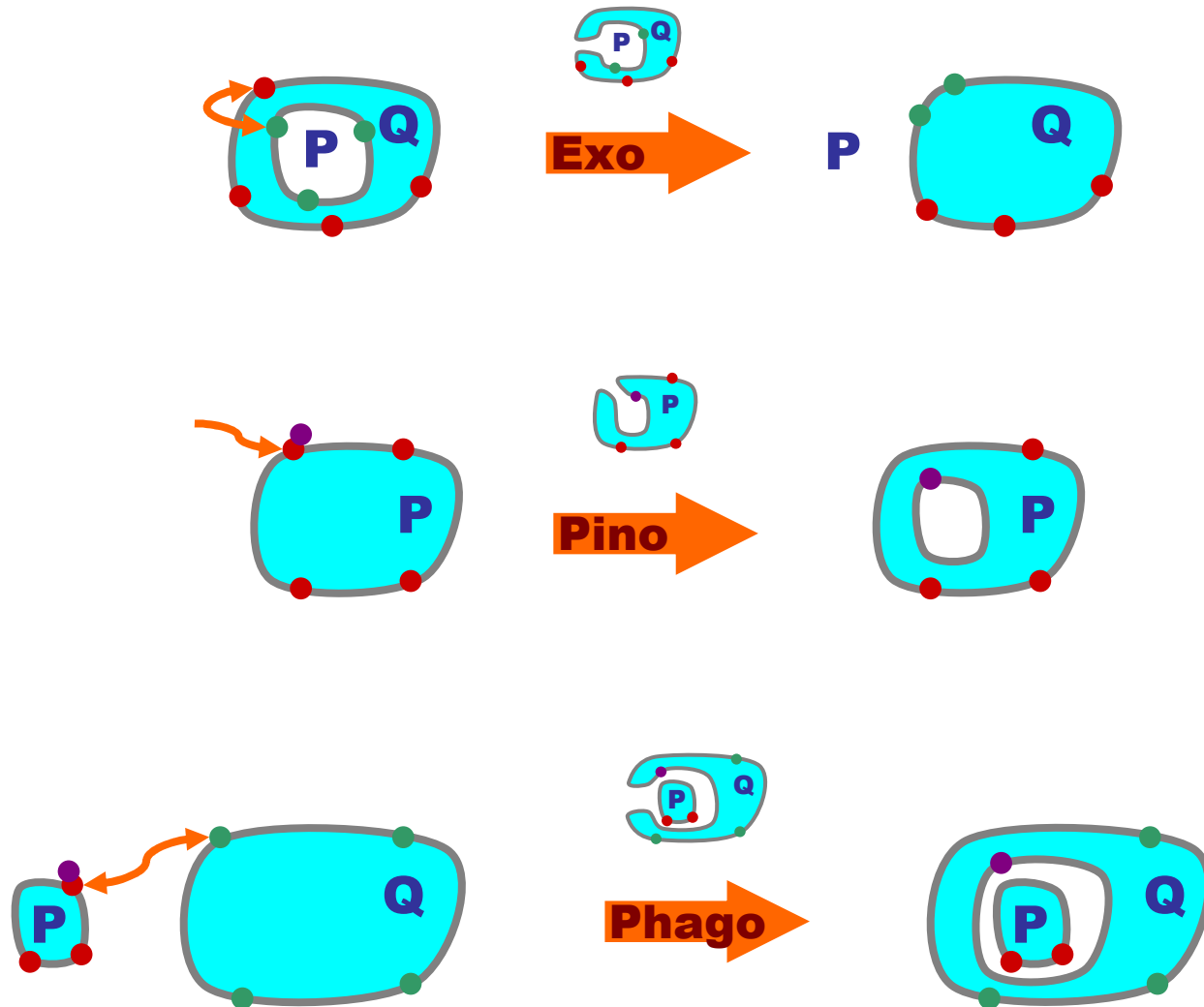
phago  $\vartheta$ , exo  $\vartheta^\perp$ , pino  $\odot$

coordination tags  
sometimes omitted



Old "spontaneous" **endo** splits into phagocytosis (**phago**, often still pronounced **endo**) and pinocytosis (**pino**).

# Brane Reactions (Cartoons)



...

**Phago**  $\vartheta_n.\sigma|\sigma'(P) \circ \vartheta_n^\perp(\rho).\tau|\tau'(Q) \longrightarrow \tau|\tau'(\rho(\sigma|\sigma'(P)) \circ Q)$

**Exo**  $\vartheta_n^\perp.\tau|\tau'(\vartheta_n.\sigma|\sigma'(P) \circ Q) \longrightarrow P \circ \sigma|\sigma'|\tau|\tau'(Q)$

**Pino**  $\curvearrowright(\rho).\sigma|\sigma'(P) \longrightarrow \sigma|\sigma'(\rho(\diamond) \circ P)$

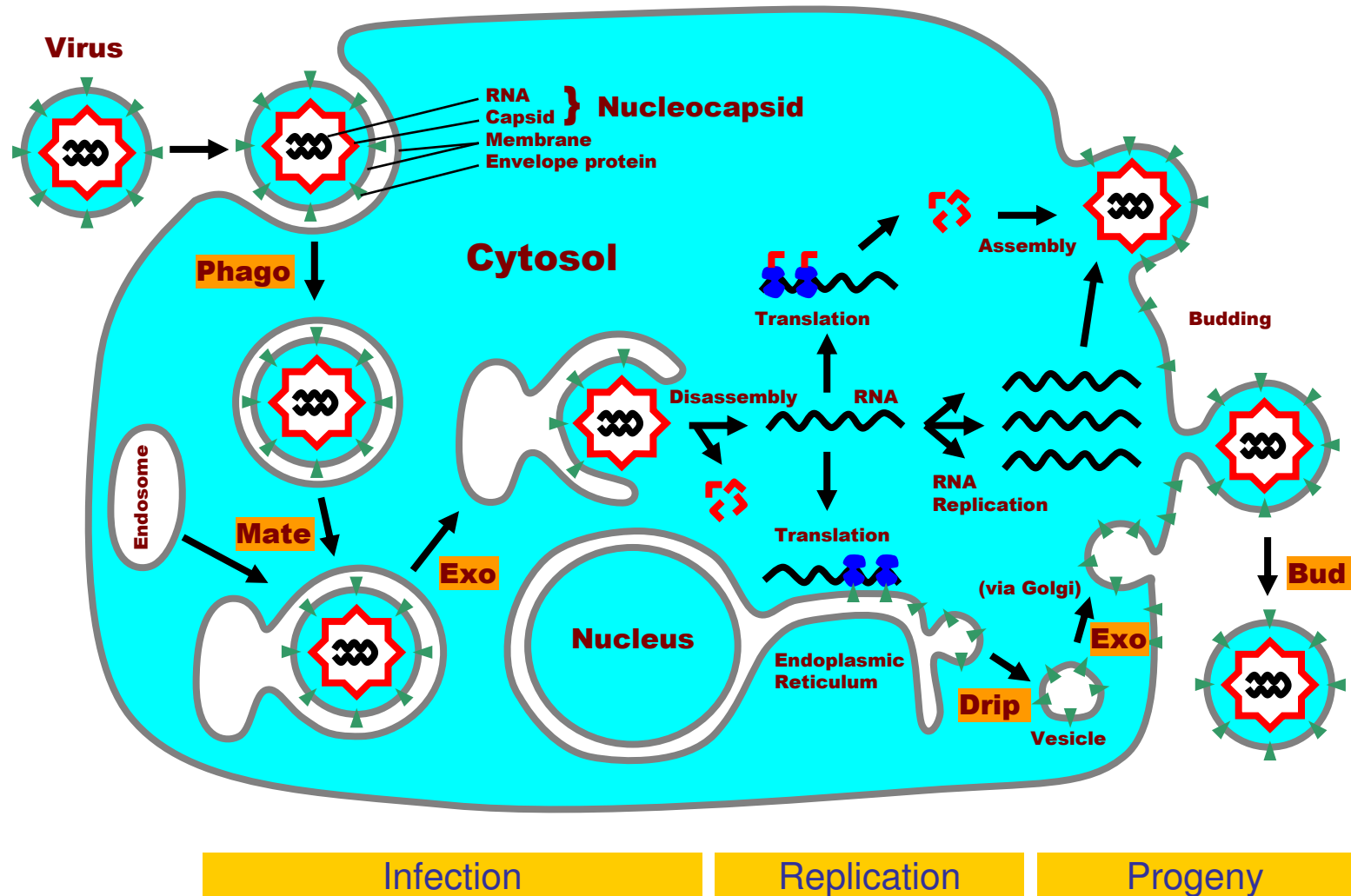
N.B.: the parity of nesting of P and Q is preserved;  
this makes the reactions preserve bitonality.

---

N.B.: in Phago (and Pino), one could perhaps require  $r$  to be, conservatively, a piece of  $t$ , by a non-linear rewrite:

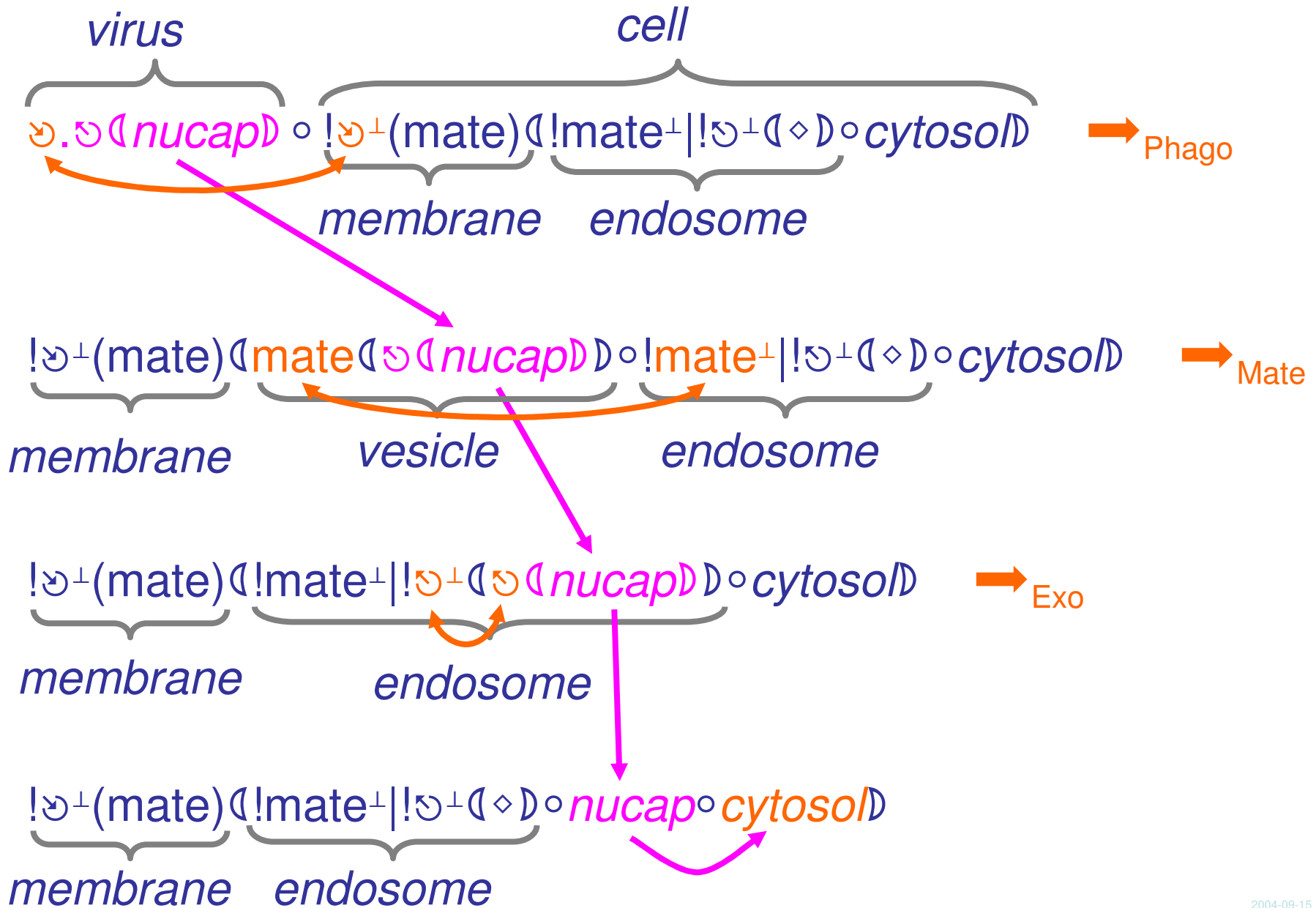
**CPhago**  $\vartheta_n.\sigma|\sigma'(P) \circ \vartheta_n^\perp(\rho).\tau|\tau'|\rho(Q) \longrightarrow \tau|\tau'(\rho(\sigma|\sigma'(P)) \circ Q)$

# Ex: Viral Reproduction





# Ex: Viral Infection

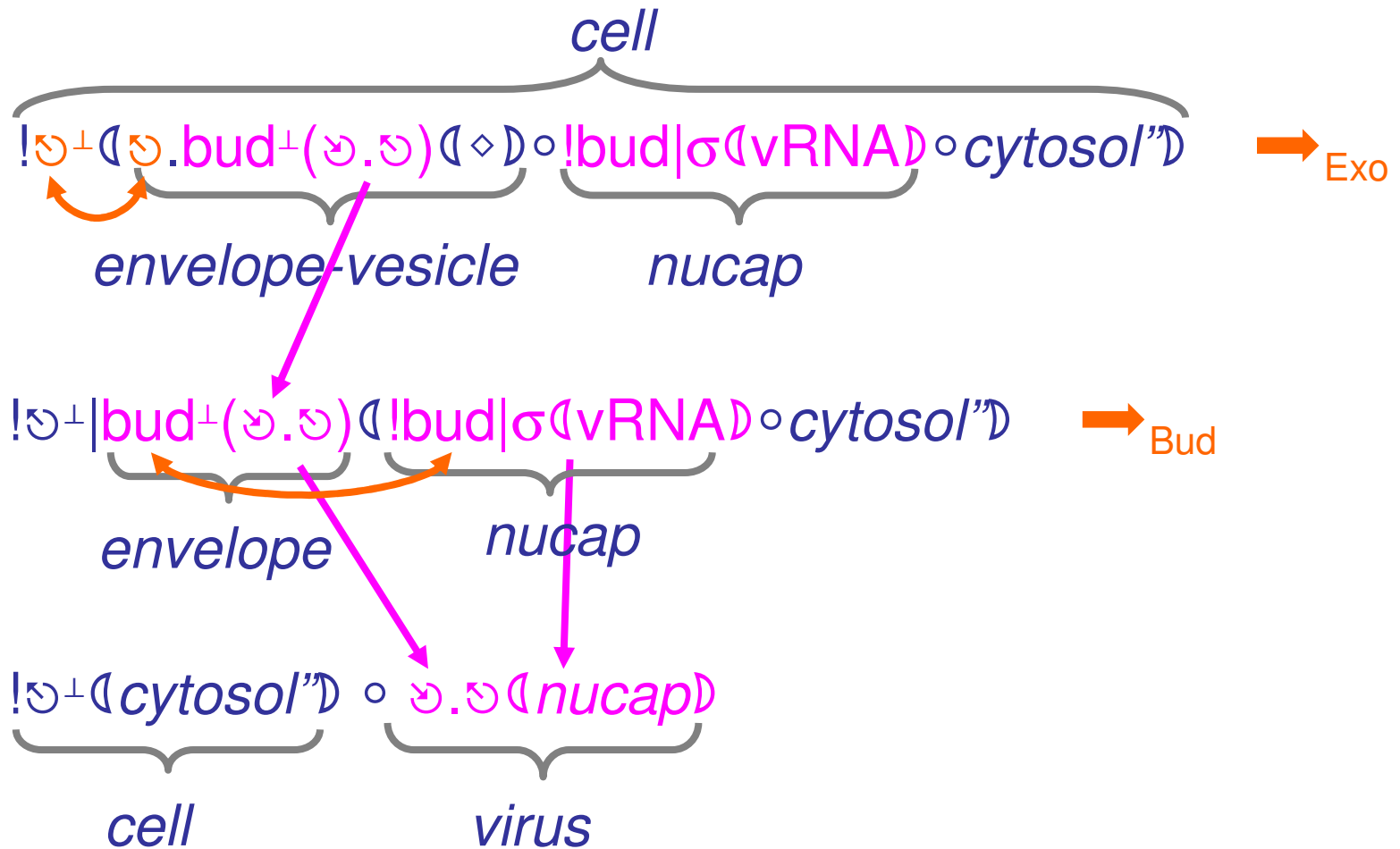


# Ex: Viral Progeny

Assume:

$nucap \circ cytosol \longrightarrow \longrightarrow nucap^n \circ envelope-vesicle^m \circ cytosol'$   
by available cellular machinery

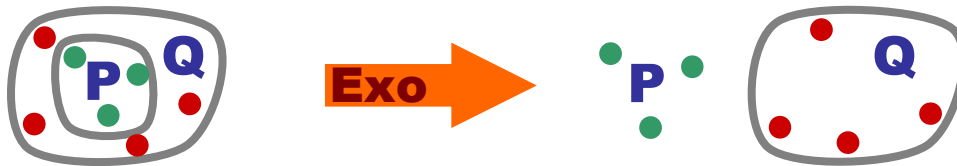
Then:



# "On Brane" vs. "In Brane"



Original "on brane"  
Exo of Brane Calculus



"In brane" encoding  
(e.g. in BioAmbients  
or SMBL) goes wrong

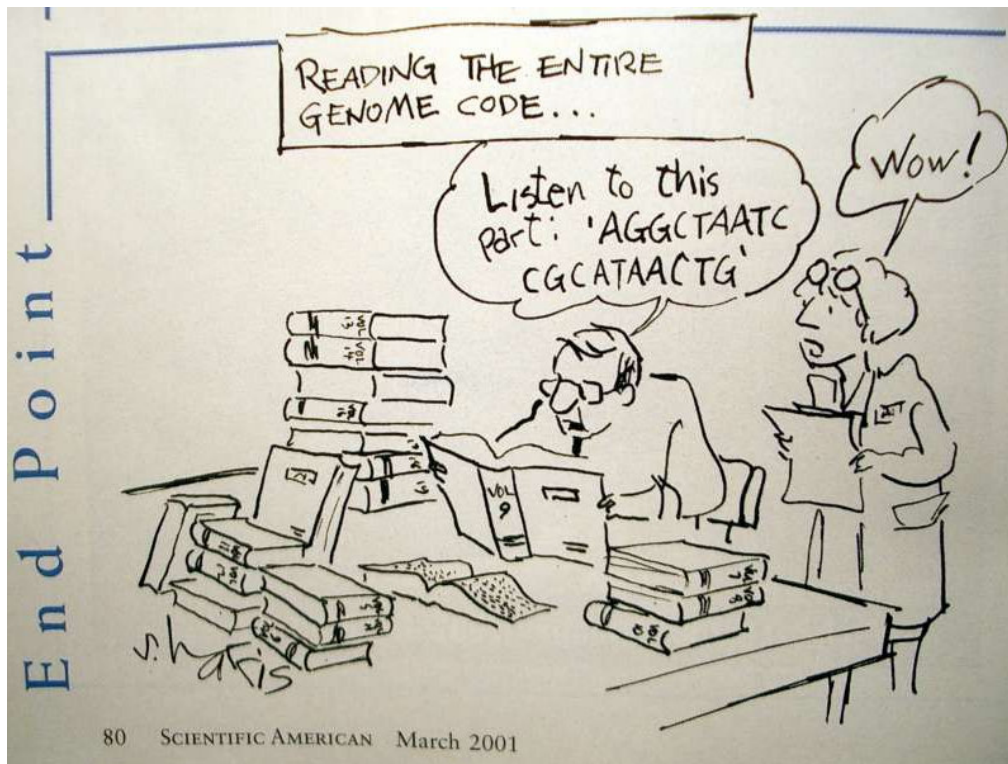


"Ball bearing"  
encoding; best we can  
do "in brane"

Awkward encoding. And all kinds of things  
can go wrong in the intermediate state.

- One cannot easily represent the Exo reaction in BioAmbients or any such compartment-based calculus, nor can one easily add it as a new primitive!
- But we can add BioAmbients-like In/Out out to Brane Calculi if we want to.

# Conclusions



**Q:** "The data are accumulating and the computers are humming, what we are lacking are **the words, the grammar and the syntax of a new language...**"

D. Bray (TIBS 22(9):325-326, 1997)

**A:** "The most advanced tools for computer process description seem to be also the best tools for the description of biomolecular systems."

E.Shapiro (Lecture Notes)

# References

[MCB] Molecular Cell Biology, Freeman.

[MBC] Molecular Biology of the Cell, Garland.

[Ptashne] A Genetic Switch.

[Davidson] Genomic Regulatory Systems.

[Milner] Communicating and Mobile Systems: the Pi-Calculus.

## Papers

### *BioAmbients*

a stochastic calculus with compartments.

### *Brane Calculi*

process calculi with computation "on" the membranes, not inside them.

### *Bitonal Systems*

membrane reactions and their connections to "local" patch reactions.

[www.luca.demon.co.uk/BioComputing.htm](http://www.luca.demon.co.uk/BioComputing.htm)